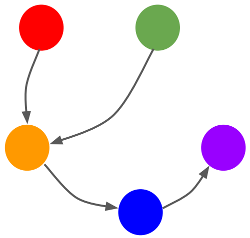
Causal Discovery



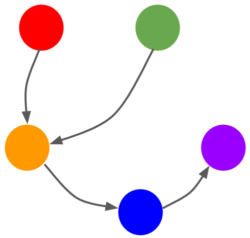
Luca Castri, PhD student @ University of Lincoln, UK

Outline

What is Causal Discovery?

Constraint-based methods
- Causal assumptions
- Markov Equivalence Class
- PC algorithm
- FCI algorithm

Causal Discovery with Time-series data
- PCMCI algorithm
- Robotics application



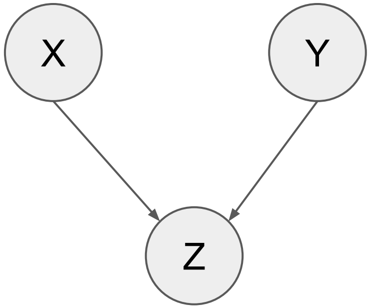
Causal Discovery

Causal Discovery: data→causal graph

	X	Y	Z
0	20.000000	100.0	340.000000
1	20.204082	100.0	340.408163
2	20.408163	100.0	340.816327
3	20.612245	100.0	341.224490
4	20.816327	100.0	341.632653
...	...	...	...
2495	29.183673	300.0	958.367347
2496	29.387755	300.0	958.775510
2497	29.591837	300.0	959.183673
2498	29.795918	300.0	959.591837
2499	30.000000	300.0	960.000000

2500 rows × 3 columns

→



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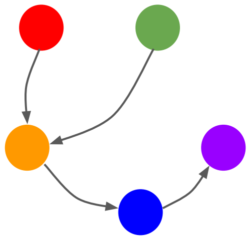

Constraint-based methods

They use conditional independence tests (constraints) to measure the independence between variables in order to identify the causal graph

PC - Peter Spirtes and **C**lark Glymour

FCI - Fast **C**ausal **I**nference

What are the assumptions?

- **Markov**
- **Faithfulness**
- **Causal Sufficiency**
- **Acyclicity**

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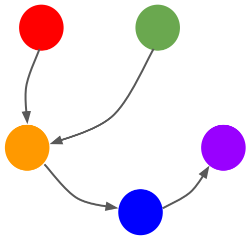


d-separation

Two (sets of) nodes X and Y are d-separated by a set of nodes Z if all the path between (any node in) X and (any node in) Y are blocked by Z

$$X \perp\!\!\!\perp_G Y | Z \quad \Rightarrow \quad X \perp\!\!\!\perp_P Y | Z$$

Markov assumption

d-separation in the graph implies conditional independencies in the distribution. **Causal graph ⇒ data**

Constraint-based methods

Causal Assumptions

d-separation

Two (sets of) nodes X and Y are d-separated by a set of nodes Z if all the path between (any node in) X and (any node in) Y are blocked by Z

$$X \perp\!\!\!\perp_G Y | Z \quad \Rightarrow \quad X \perp\!\!\!\perp_P Y | Z$$

Markov assumption

d-separation in the graph implies conditional independencies in the distribution. **Causal graph ⇒ data**

$$X \perp\!\!\!\perp_G Y | Z \quad \Leftarrow \quad X \perp\!\!\!\perp_P Y | Z$$

Faithfulness

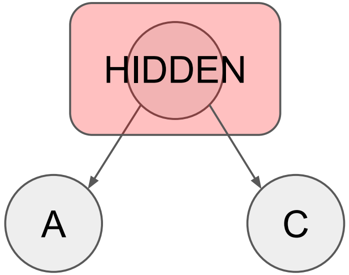
To do causal discovery we need to go the other way around.
Causal graph ⇐ data

It is the inverse of the Markov assumption and assesses:

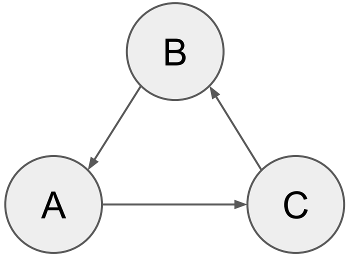
if two variables X and Y are independent conditioning on Z in the distribution then X and Y are d-separated by Z in the causal graph

Constraint-based methods
Causal Assumptions

Causal Sufficiency
There are no unobserved confounders of any of the variables in the graph



Acyclicity
No cycle in the graph



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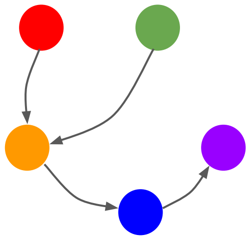
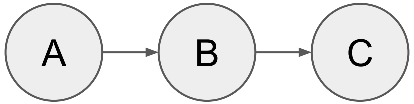
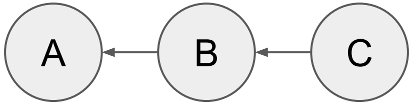


Constraint-based methods
Markov Equivalence Class

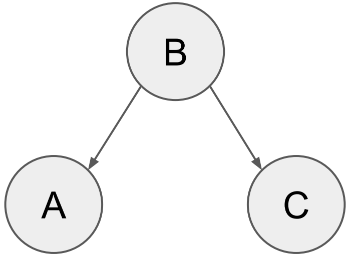
Chain



Chain



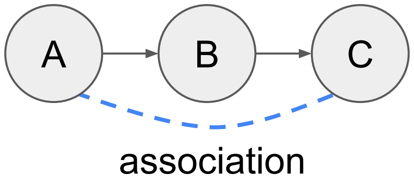
Fork



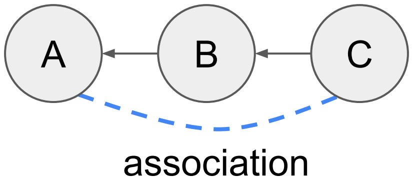
Constraint-based methods

Markov Equivalence Class

Chain



Chain



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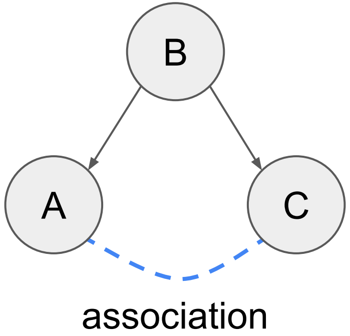
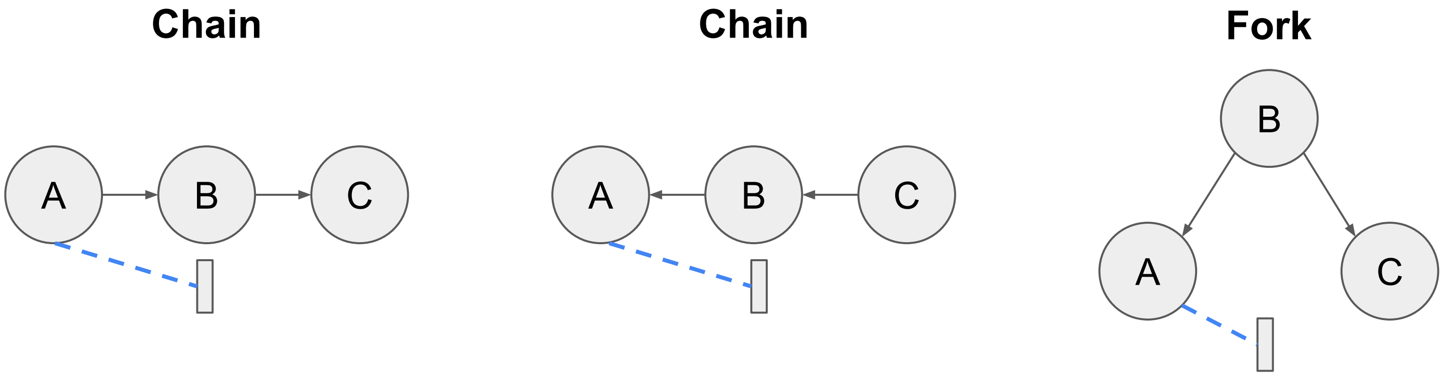


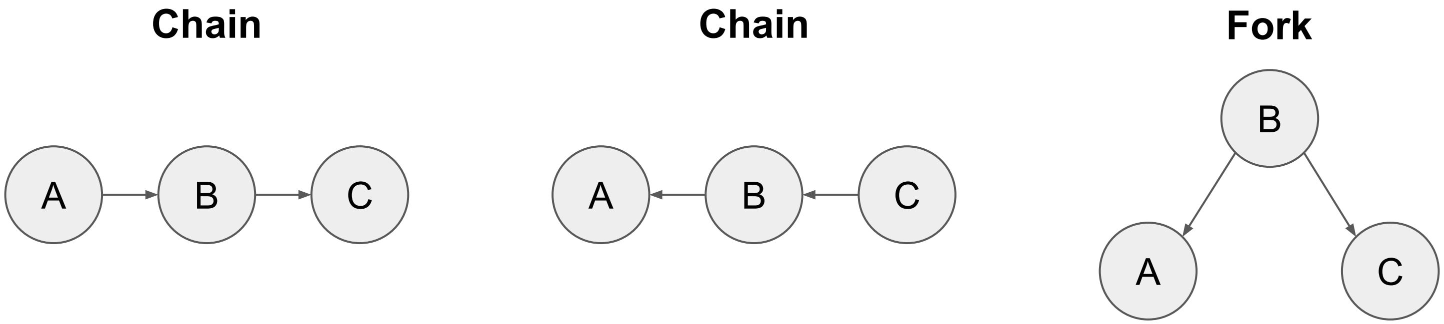
Constraint-based methods
Markov Equivalence Class



Markov: $A \perp\!\!\!\perp C | B$ They all imply the same conditional independence

Constraint-based methods

Markov Equivalence Class



Markov equivalent
They all belong to the same **Markov Equivalence Class**

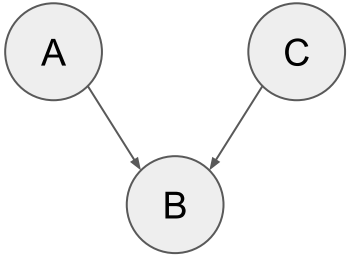
Markov: $A \perp\!\!\!\perp C | B$ They all imply the same conditional independence

Constraint-based methods

Markov Equivalence Class

Markov: $A \perp\!\!\!\perp C$

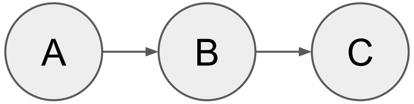
Collider

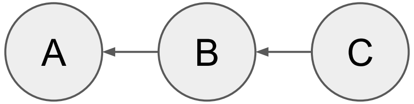


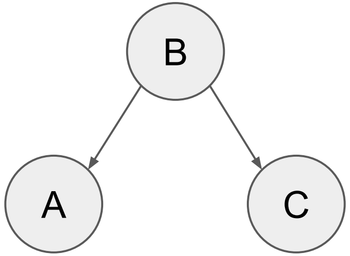
Constraint-based methods

Markov Equivalence Class

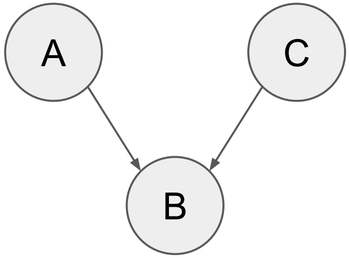
Markov: $A \perp\!\!\!\perp C \mid B$







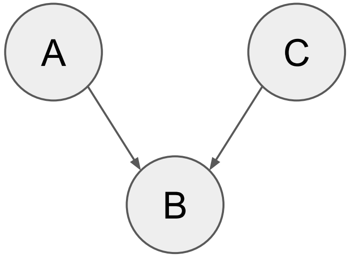
Markov: $A \perp\!\!\!\perp C$



Constraint-based methods

Markov Equivalence Class

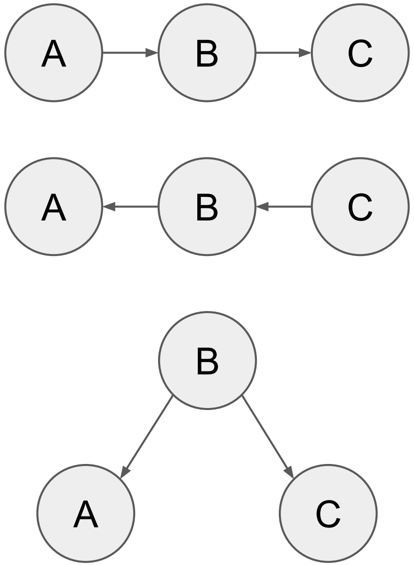
Markov: $A \perp\!\!\!\perp C$



If we find a triple of variables which satisfies this Markov assumption
$\Rightarrow$ collider configuration

COLLIDERS ARE IMPORTANT

Markov: $A \perp\!\!\!\perp C | B$



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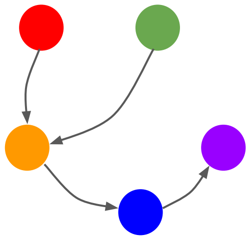

Constraint-based methods

PC algorithm

Markov | Faithfulness | Causal Sufficiency | Acyclicity

1. Start with fully connected and undirected graph
2. Identify the skeleton
3. Identify colliders and orient them
4. Orient the non-colliders edges (orientation propagation)

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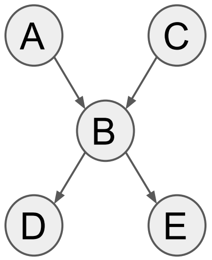

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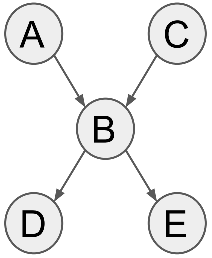

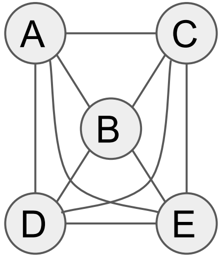

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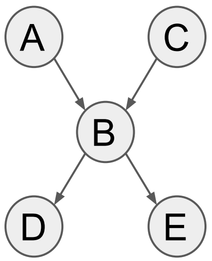

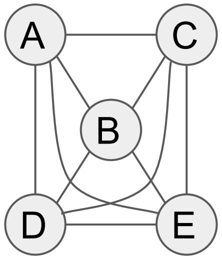

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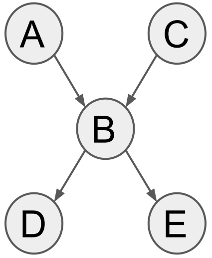
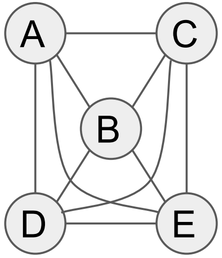
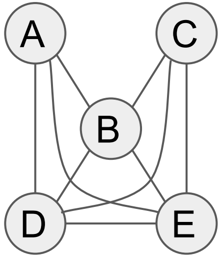


$A \perp\!\!\!\perp C | \{\}$





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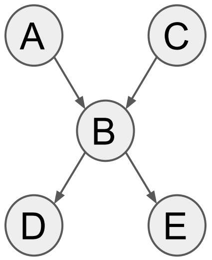
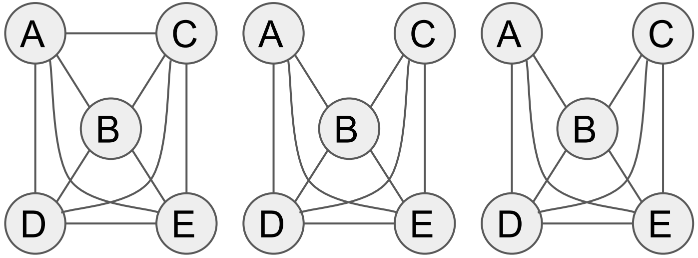


$A \perp\!\!\!\perp C | \{\}$



$A \perp\!\!\!\perp D | \{\}$ ✖
$D \perp\!\!\!\perp E | \{\}$ ✖
$C \perp\!\!\!\perp E | \{\}$ ✖
$A \perp\!\!\!\perp E | \{\}$ ✖
$C \perp\!\!\!\perp D | \{\}$ ✖

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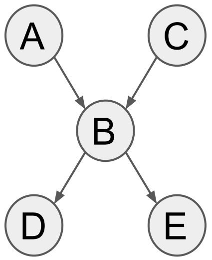
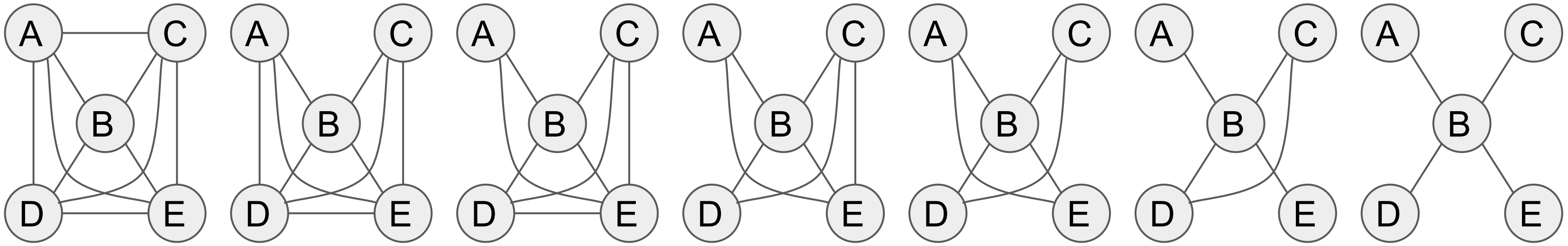


$A \perp\!\!\!\perp C|\{\}$ $\quad A \perp\!\!\!\perp D|\{B\}$ $\quad D \perp\!\!\!\perp E|\{B\}$ $\quad C \perp\!\!\!\perp E|\{B\}$ $\quad A \perp\!\!\!\perp E|\{B\}$ $\quad C \perp\!\!\!\perp D|\{B\}$



Constraint-based methods

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1. Start with fully connected and undirected graph
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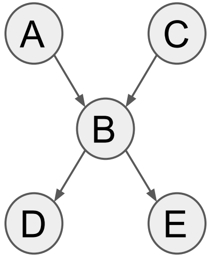

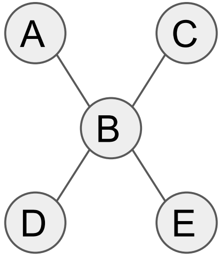

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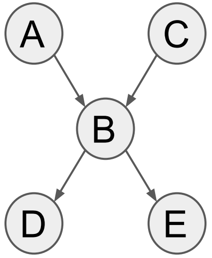
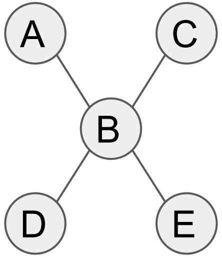
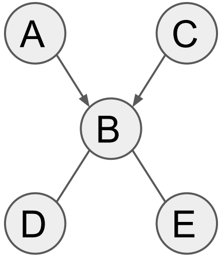


$A \perp\!\!\!\perp C | \{\}$ We removed A – C with conditioning set empty
⇒ unique markov equivalence class: **collider**
⇒ A→B←C





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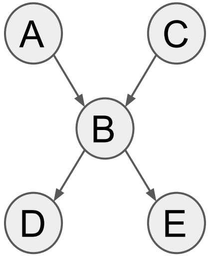

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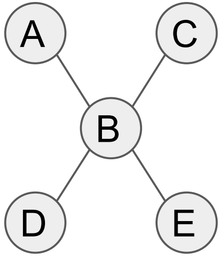
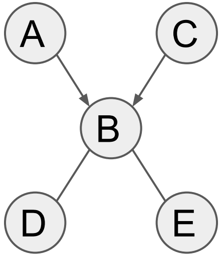
$A \perp\!\!\!\perp D | \{B\}$ $A \perp\!\!\!\perp E | \{B\}$ $D \perp\!\!\!\perp E | \{B\}$ $C \perp\!\!\!\perp D | \{B\}$ $C \perp\!\!\!\perp E | \{B\}$

We removed all these edges conditioning on B
⇒ markov equivalence class: **chain** or **fork**





Constraint-based methods
PC algorithm

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1. Start with fully connected and undirected graph
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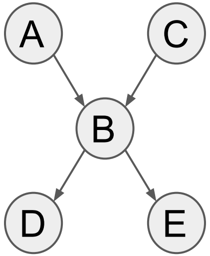
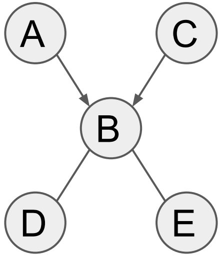
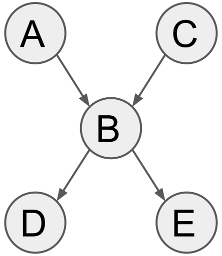
4. **Orient the non-colliders edges (orientation propagation)**
   any edge $Z – Y$ part of a partially directed path $X \rightarrow Z – Y$, where there is no edge between X and Y can be oriented as $Z \rightarrow Y$





$A \rightarrow B – D$, no edge between A and D $\Rightarrow A \rightarrow B \rightarrow D$
$A \rightarrow B – E$, no edge between A and E $\Rightarrow A \rightarrow B \rightarrow E$



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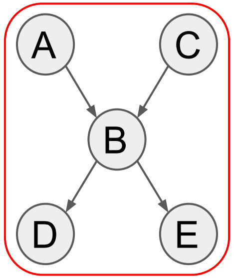
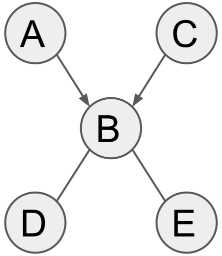
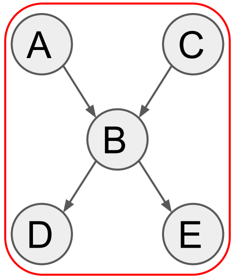




A→B – D, no edge between A and D ⇒ A→B→D
A→B – E, no edge between A and E ⇒ A→B→E



Outline

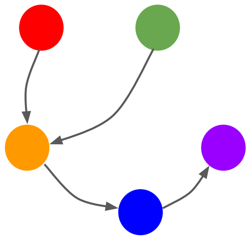
What is Causal Discovery?

Constraint-based methods

- Causal assumptions
- Markov Equivalence Class
- PC algorithm
- **FCI algorithm**

Causal Discovery with Time-series data

- PCMCI algorithm
- Robotics application

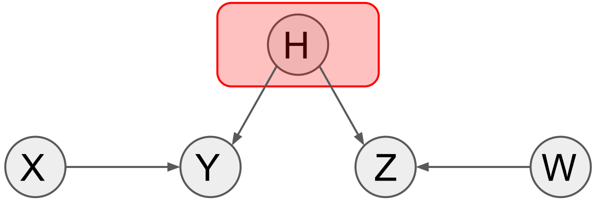


Constraint-based methods

FCI algorithm

Markov | Faithfulness | Causal Sufficiency | Acyclicity

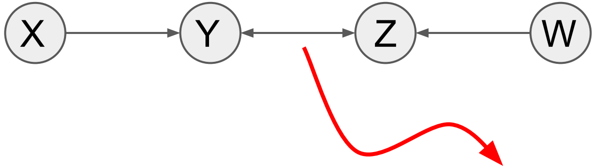
variation of the PC algorithm which tolerates hidden confounders.



skeleton

colliders identification

colliders identification



The bidirected edge indicates that there is at least one <u>hidden confounders</u>

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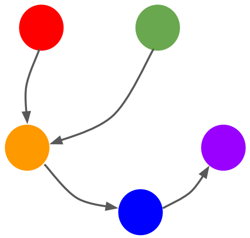
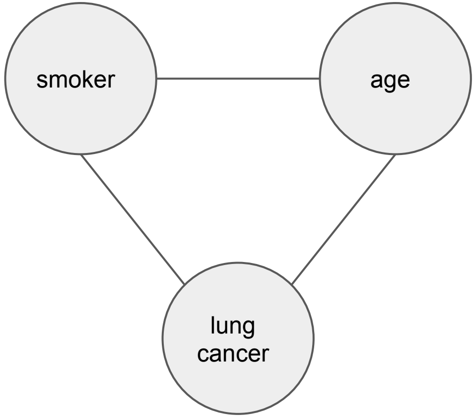


Causal Discovery with Time-series data

The PC and FCI causal discovery method work well with discrete/categorical data.

example
non-smoker **0** - smoker **1**
age under 50 **0** - age over 50 **1**
no lung cancer **0** - lung cancer **1**

Smoker	Age	Lung cancer
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



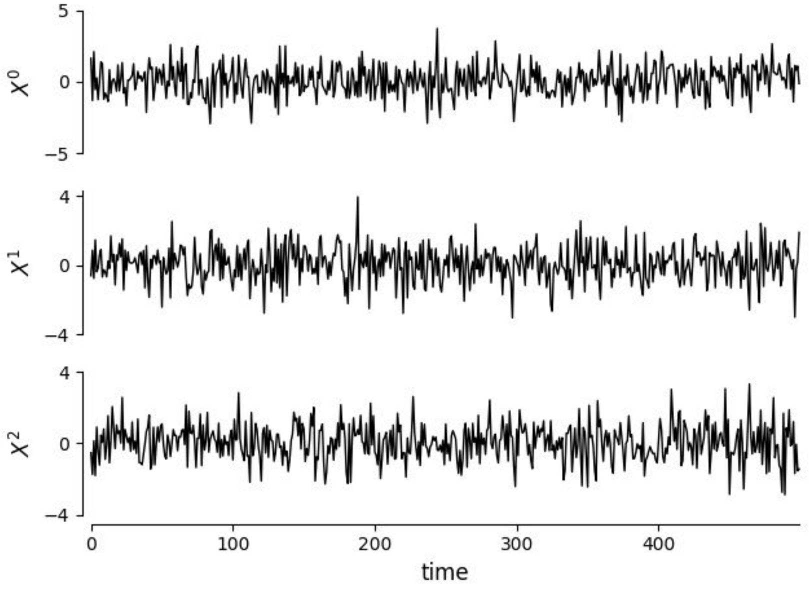
Causal Discovery with Time-series data

The PC and FCI causal discovery method work well with discrete/categorical data.

What if we deal with time-series data?
PC is inappropriate to use with time series data due to:

- time ordering
- lagged dependencies
- high false positive rates due to the autocorrelation.

$$\begin{cases} X_t^0 = 0.2(X_{t-1}^1)^2 + \eta_t^0 \\ X_t^1 = \eta_t^1 \\ X_t^2 = 0.3(X_{t-2}^1)^2 + \eta_t^2 \end{cases}$$



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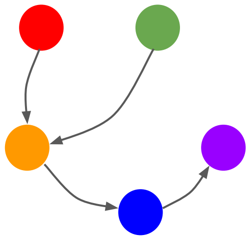

Causal Discovery with Time-series data

PCMCI algorithm

It consists of two main steps:

- **PC algorithm**
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- **MCI test**
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t | \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$

Key parameter: τ maximum time delay

Causal Discovery with Time-series data
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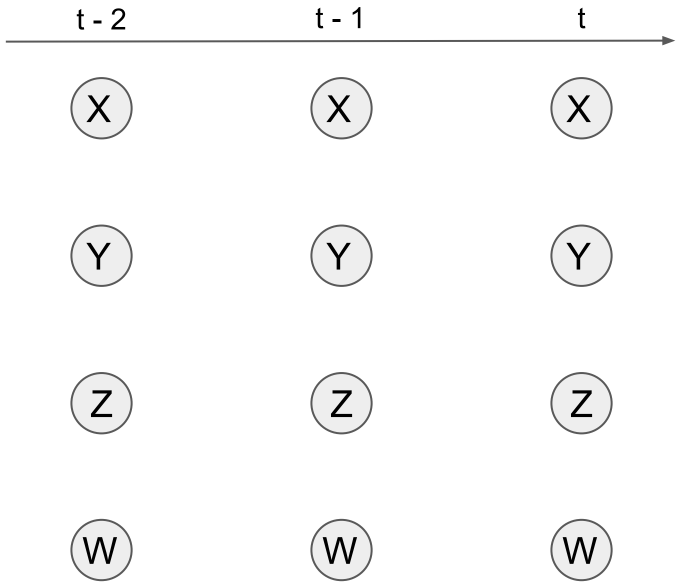

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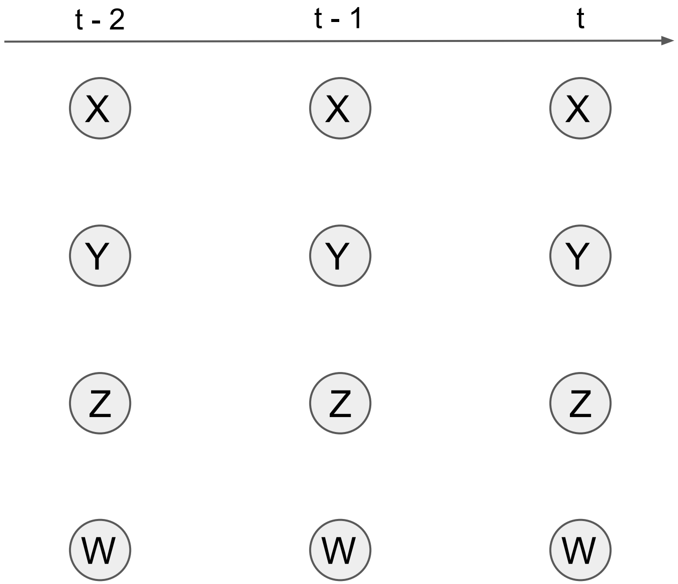

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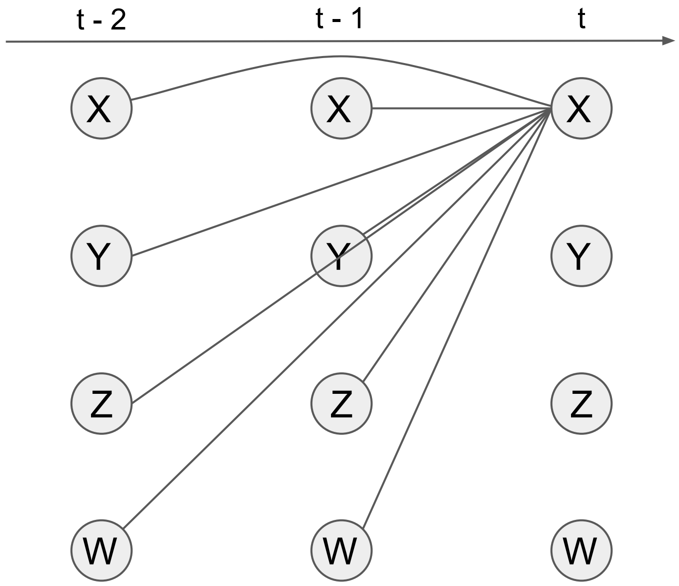

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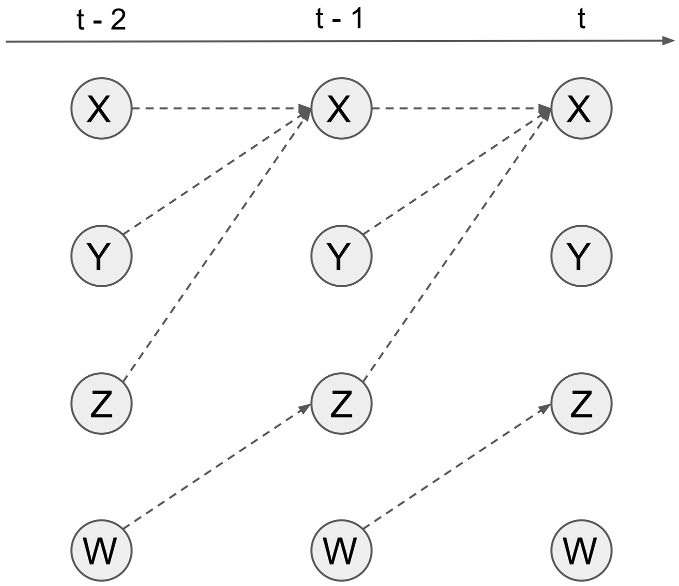

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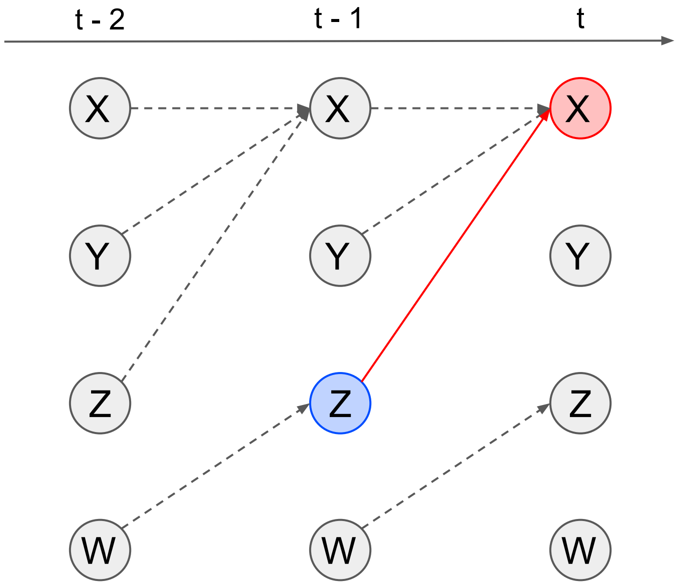

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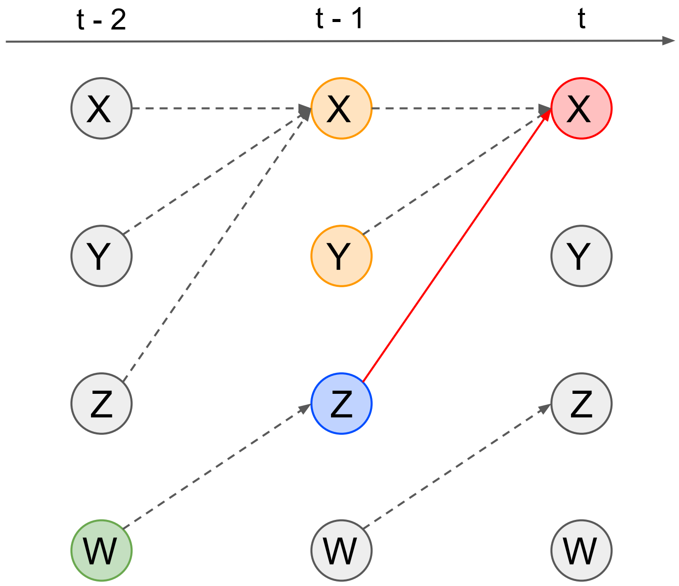
Key parameter: τ maximum time delay



Causal Discovery with Time-series data

PCMCI algorithm

$$\begin{cases} X_t^0 = 0.4(X_{t-1}^1)^2 + \eta_t^0 \\ X_t^1 = \eta_t^1 \\ X_t^2 = 0.3(X_{t-2}^1)^2 + \eta_t^2 \end{cases}$$

```
random_state = np.random.default_rng(seed=42)
data = random_state.standard_normal((500, 3))
for t in range(1, 500):
    data[t, 0] += 0.4*data[t-1, 1]**2
    data[t, 2] += 0.3*data[t-2, 1]**2
var_names = [r'$X^0$', r'$X^1$', r'$X^2$']

dataframe = pp.DataFrame(data, var_names=var_names)
```

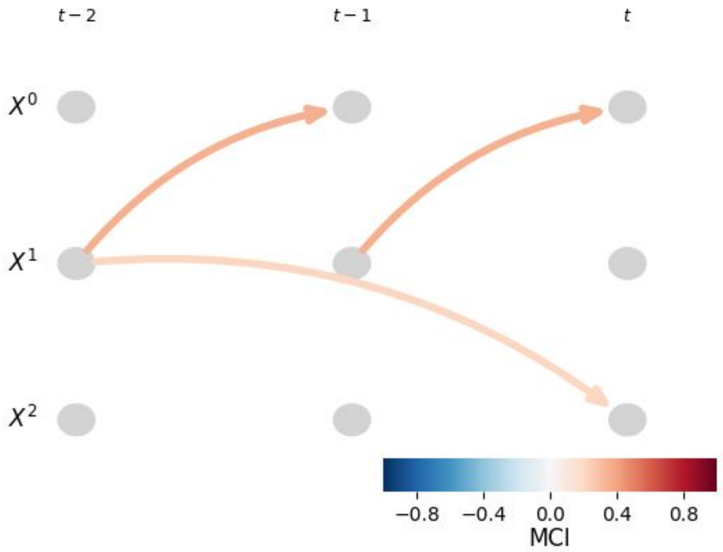
```
gpdc = GPDC(significance='analytic', gp_params=None)
pcmci_gpdc = PCMCI(
    dataframe=dataframe,
    cond_ind_test=gpdc,
    verbosity=0)
```

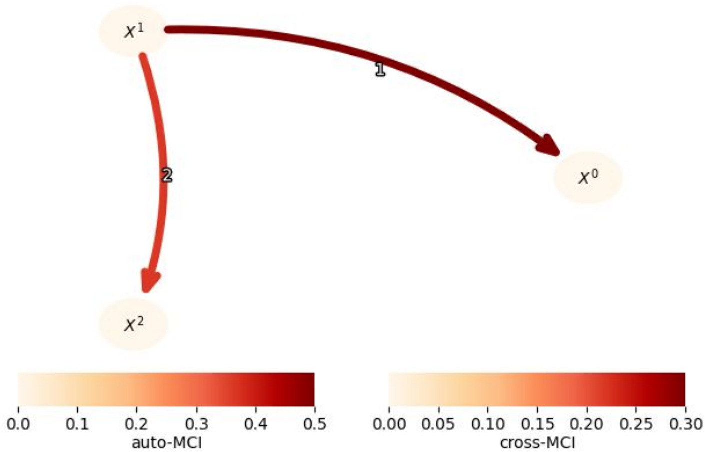
```
results = pcmci_gpdc.run_pcmci(tau_max=2, pc_alpha=0.1, alpha_level = 0.01)
tp.plot_graph(
    val_matrix=results['val_matrix'],
    graph=results['graph'],
    var_names=var_names,
    show_colorbar=False,
    )
```

Causal Discovery with Time-series data

PCMCI algorithm

$$
\begin{cases}
X_t^0 = 0.4(X_{t-1}^1)^2 + \eta_t^0 \\
X_t^1 = \eta_t^1 \\
X_t^2 = 0.3(X_{t-2}^1)^2 + \eta_t^2
\end{cases}
$$





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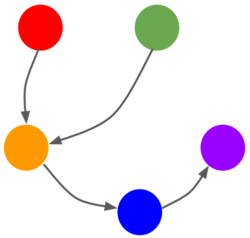
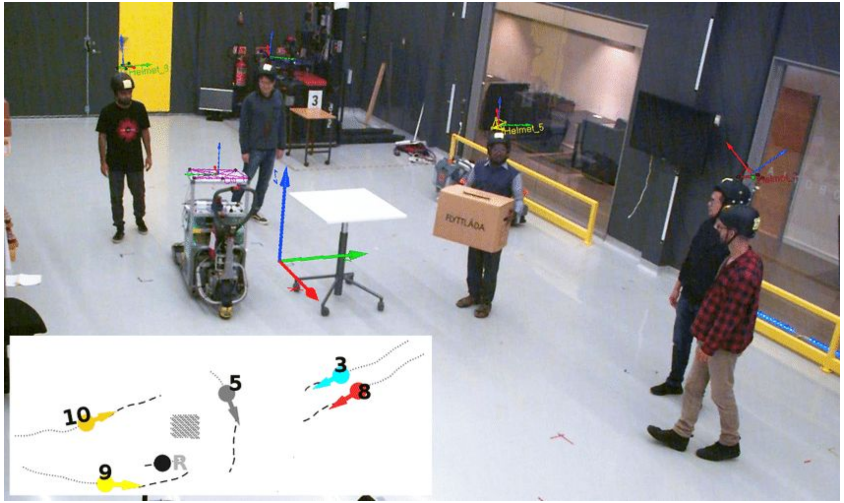
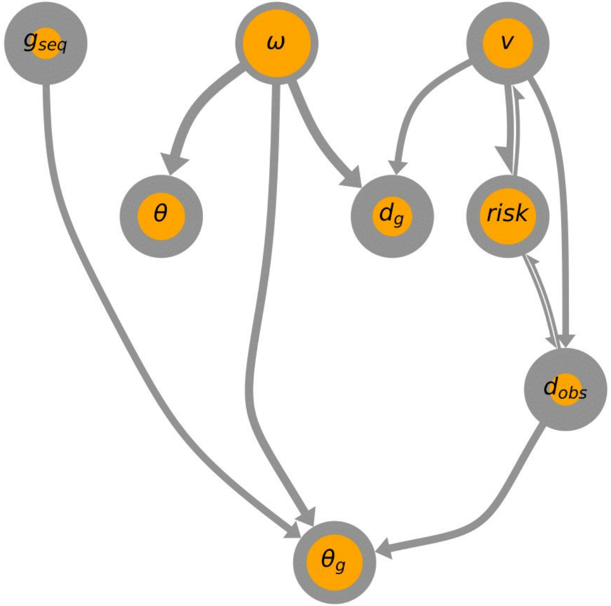


Causal Discovery with Time-series data

Robotics application

Only observational data



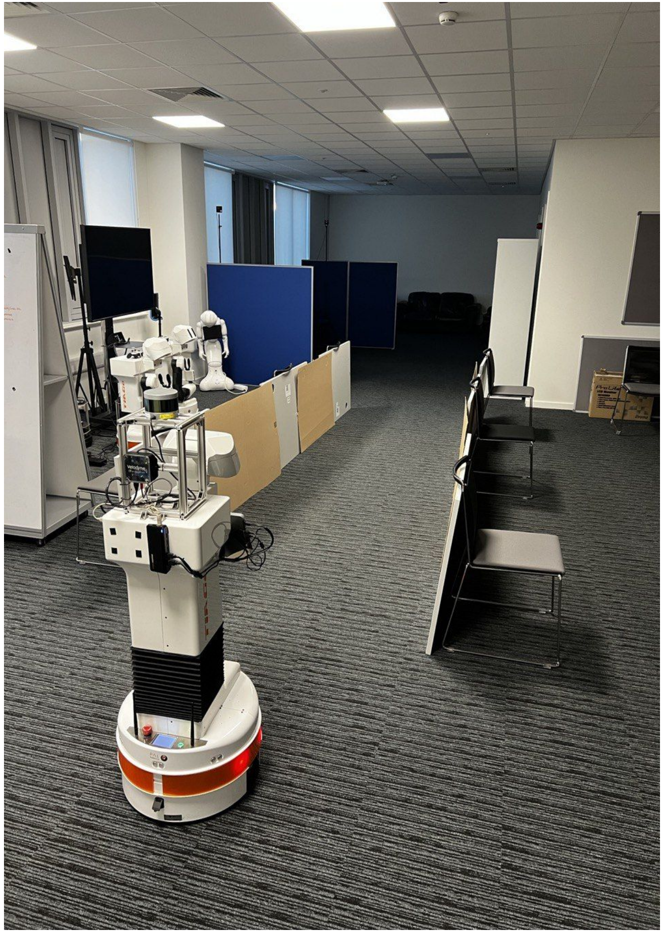


Castri, Luca, et al. "Causal Discovery of Dynamic Models for Predicting Human Spatial Interactions." (2022).
Castri, Luca, et al. "Enhancing Causal Discovery from Robot Sensor Data in Dynamic Scenarios." (2023).

Causal Discovery with Time-series data
Robotics application

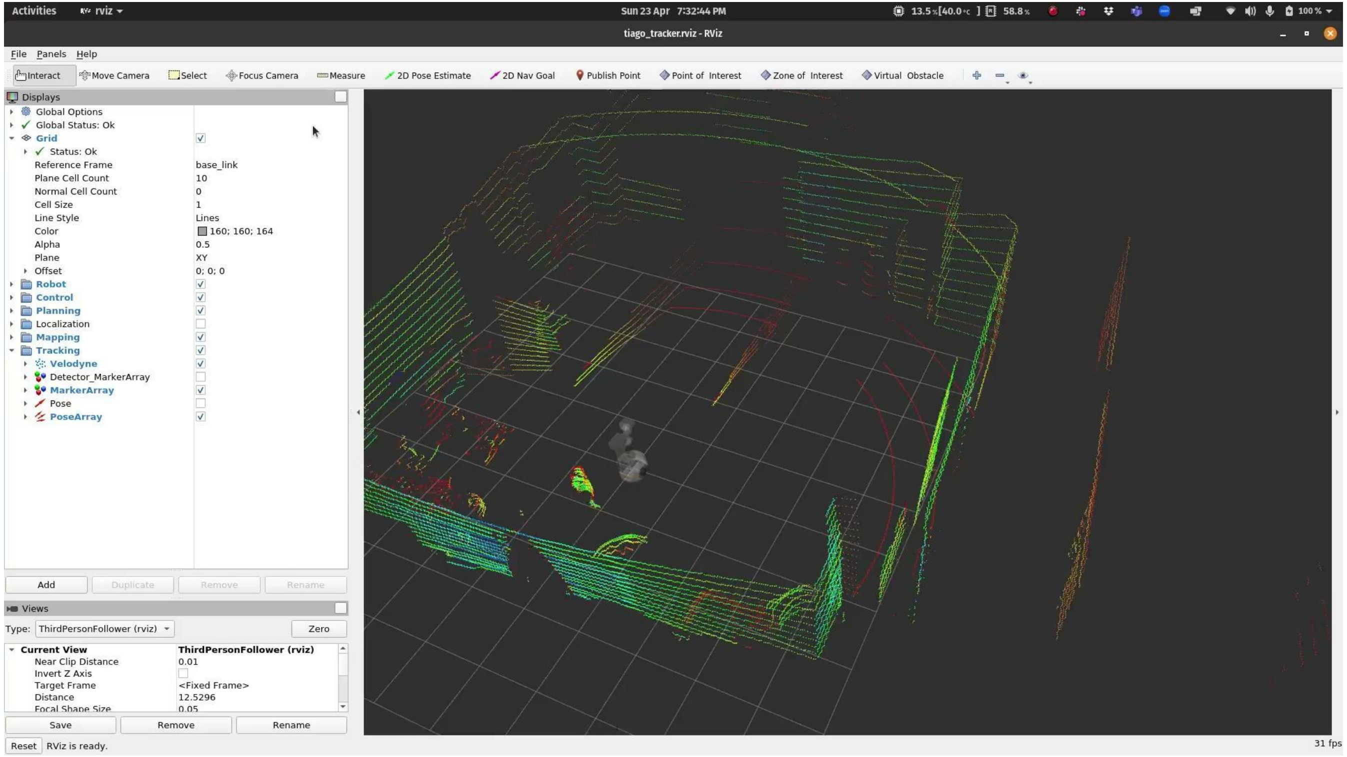
Observational and interventional data
The experiment consists of:

- TIAGo robot
- a human
- a “corridor”
- human overtakes the robot
  the robot tries to perform an action (intervention) to facilitate the overtake



Causal Discovery with Time-series data

Robotics application



CAUSAL MODEL

?

still ongoing..

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Thank you questions?