Causal Discovery for Time-Series Data

Luca Castri, PhD student @ University of Lincoln, UK

Outline

- **Causal Discovery for Time-series Data**
  - PCMCI algorithm
- Robotics Applications
  - F-PCMCI algorithm
  - ROS-Causal

Causal Discovery for Time-series Data

The PC and FCI causal discovery method work well with discrete/categorical data.

example

non-smoker **0** - smoker **1**

age under 50 **0** - age over 50 **1**

no lung cancer **0** - lung cancer **1**

Smoker	Age	Lung cancer
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
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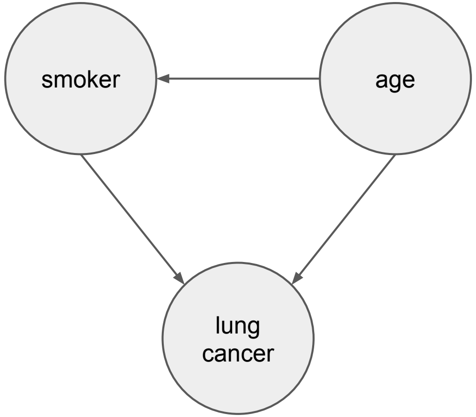

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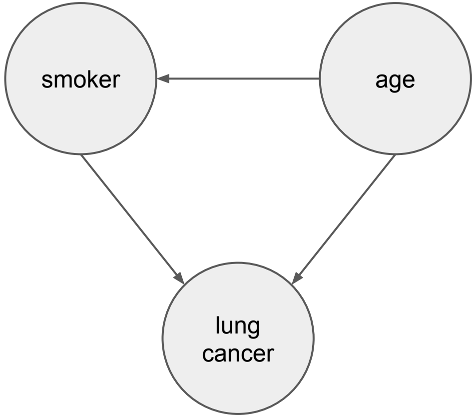

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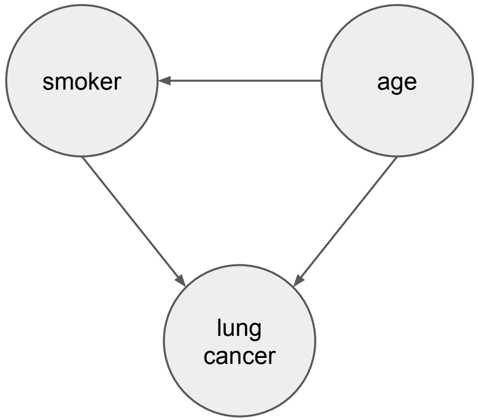

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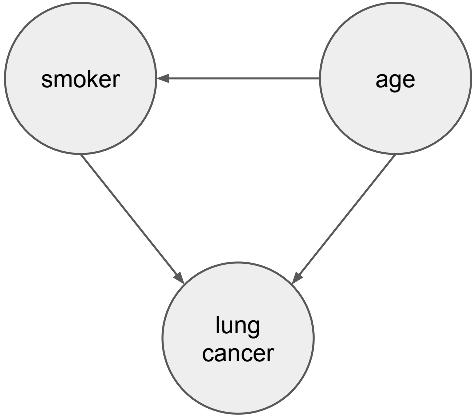

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The order is not important

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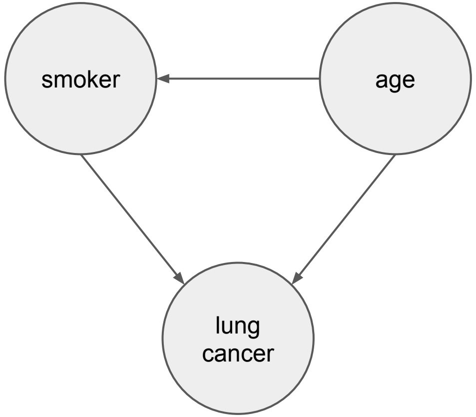
What if our data is time-dependent?



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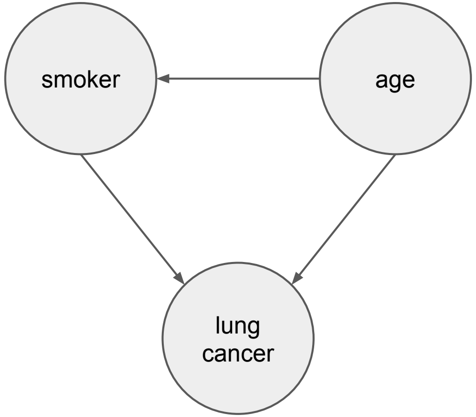

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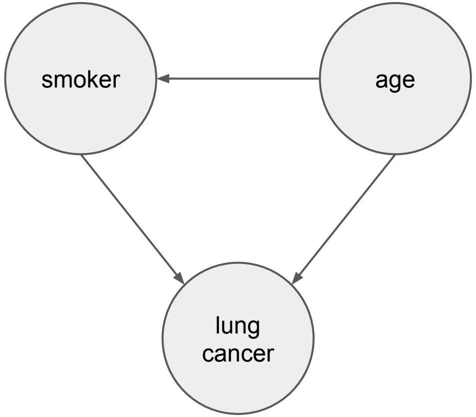

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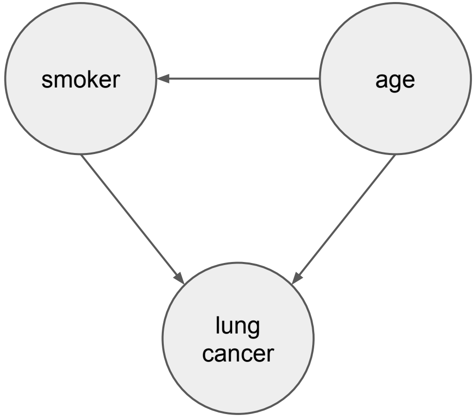

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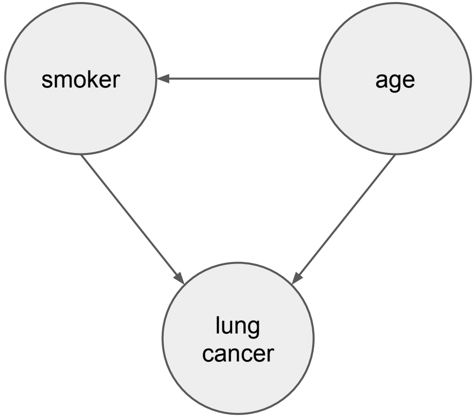

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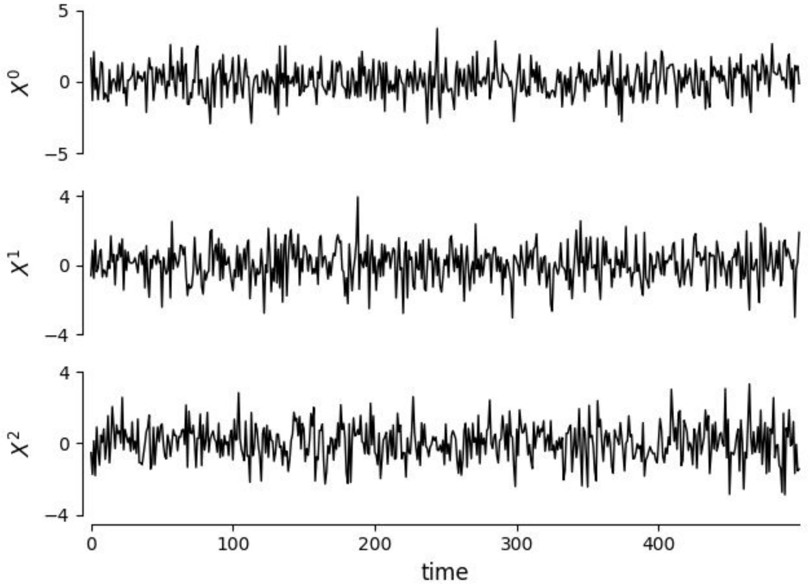
Causal Discovery for Time-series Data

The PC and FCI causal discovery method work well with discrete/categorical data.

What if we deal with time-series data?
PC/FCI is inappropriate to use with time series data due to:

- time ordering
- lagged dependencies
- high false positive rates due to the autocorrelation.

$$\begin{cases} X_t^0 = 0.2(X_{t-1}^1)^2 + \eta_t^0 \\ X_t^1 = \eta_t^1 \\ X_t^2 = 0.3(X_{t-2}^1)^2 + \eta_t^2 \end{cases}$$



Outline

- Causal Discovery for Time-series Data
  - **PCMCI algorithm**
- Robotics Applications
  - F-PCMCI algorithm
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Causal Discovery for Time-series Data

PCMCI algorithm

It consists of two main steps:

- **PC algorithm**
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- **MCI test**
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t | \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$

Key parameter: τ maximum time delay

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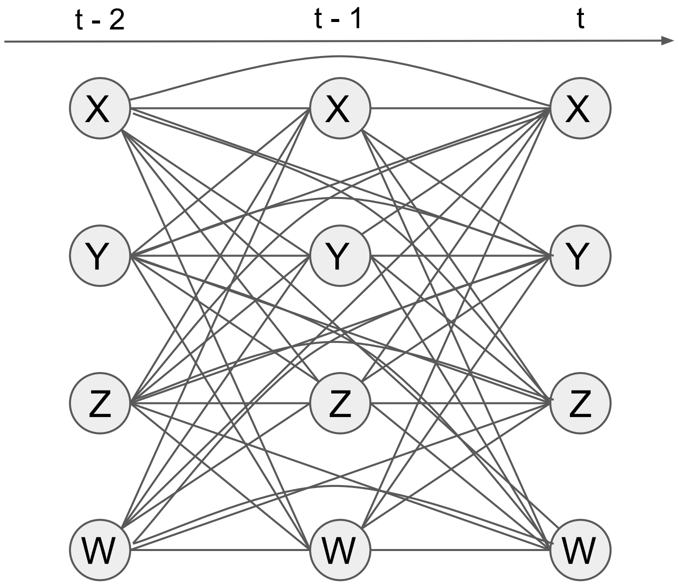

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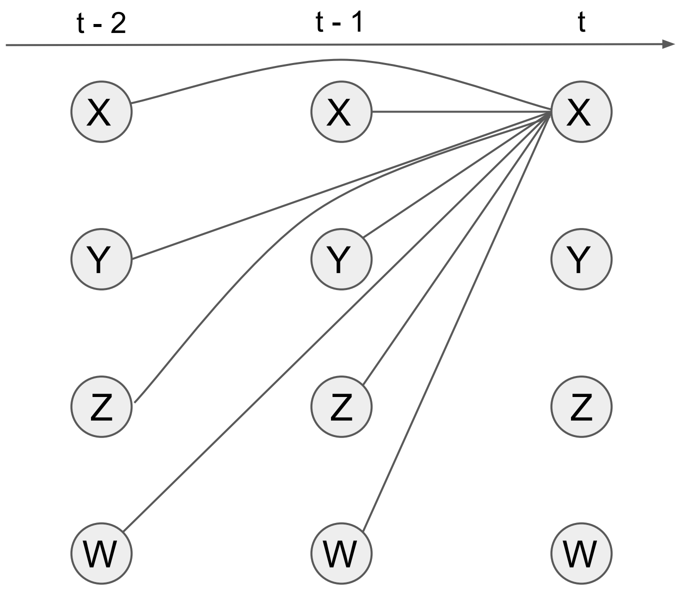

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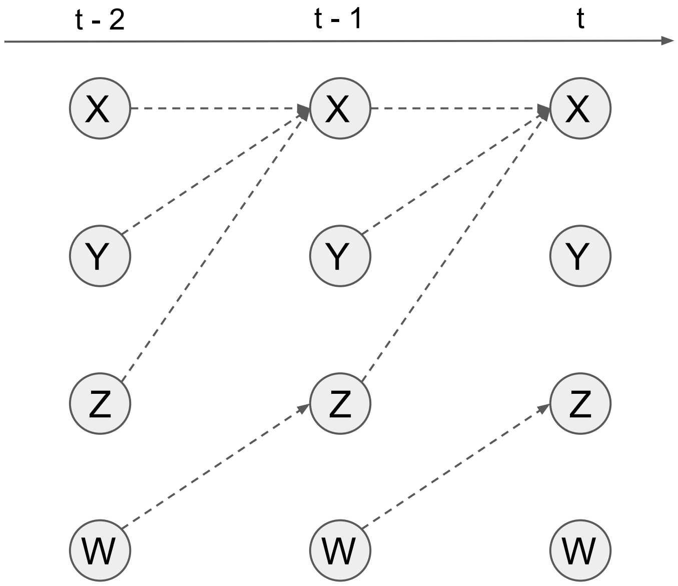

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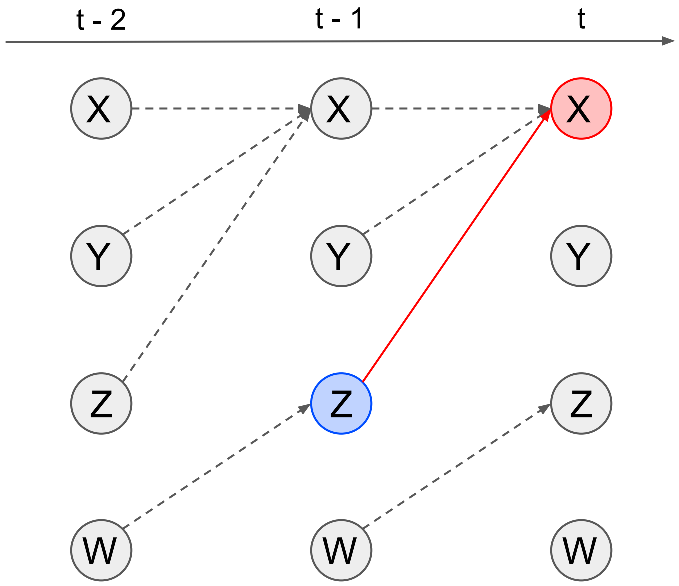

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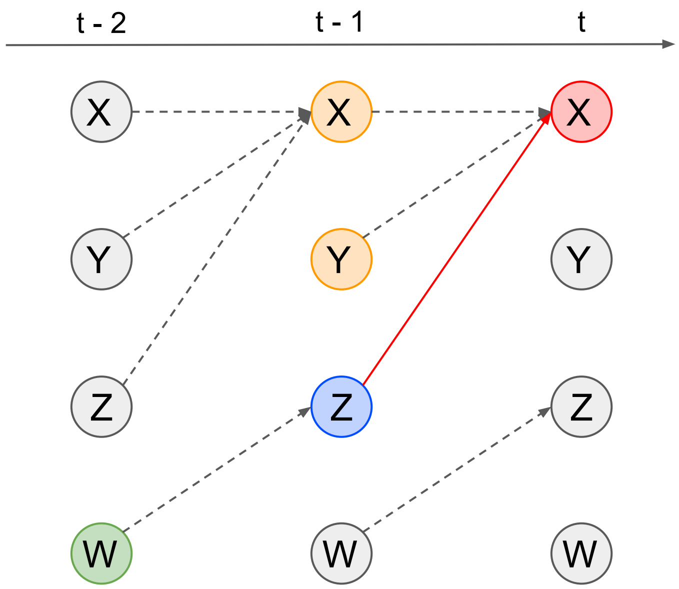

Causal Discovery for Time-series Data

PCMCI algorithm

$$\begin{cases} X_t^0 = 0.4(X_{t-1}^1)^2 + \eta_t^0 \\ X_t^1 = \eta_t^1 \\ X_t^2 = 0.3(X_{t-2}^1)^2 + \eta_t^2 \end{cases}$$

```
random_state = np.random.default_rng(seed=42)
data = random_state.standard_normal((500, 3))
for t in range(1, 500):
    data[t, 0] += 0.4*data[t-1, 1]**2
    data[t, 2] += 0.3*data[t-2, 1]**2
var_names = [r'$X^0$', r'$X^1$', r'$X^2$']

dataframe = pp.DataFrame(data, var_names=var_names)
```

```
gpdc = GPDC(significance='analytic', gp_params=None)
pcmci_gpdc = PCMCI(
    dataframe=dataframe,
    cond_ind_test=gpdc,
    verbosity=0)
```

```
results = pcmci_gpdc.run_pcmci(tau_max=2, pc_alpha=0.1, alpha_level = 0.01)
tp.plot_graph(
    val_matrix=results['val_matrix'],
    graph=results['graph'],
    var_names=var_names,
    show_colorbar=False,
    )
```

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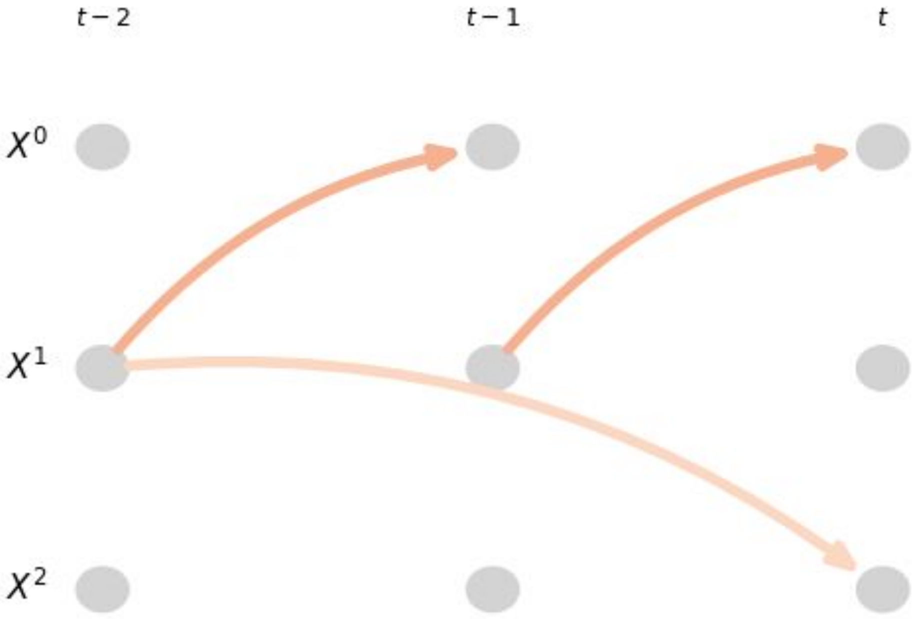

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- Causal Discovery for Time-series Data
  - PCMCI algorithm

- **Robotics Applications**
  - F-PCMCI algorithm
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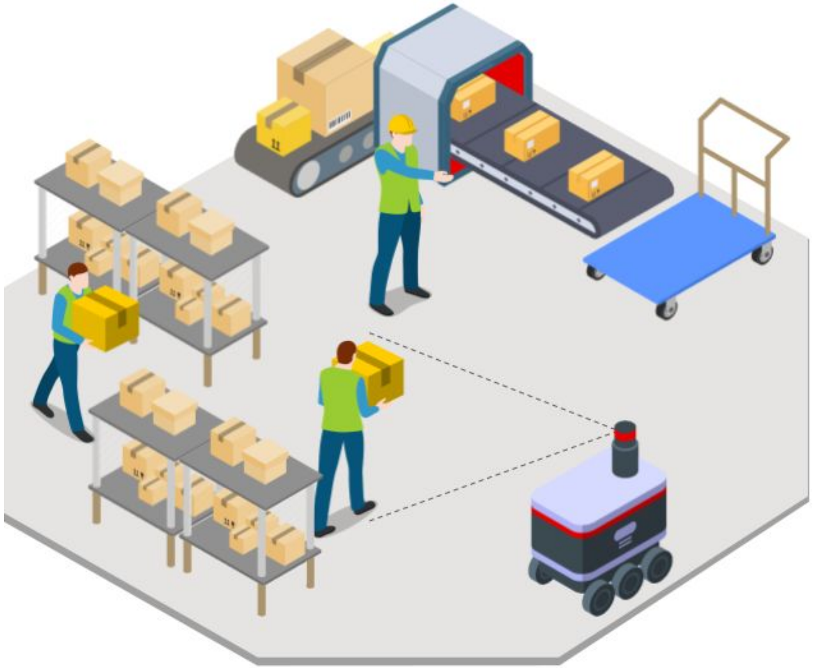
Robotics Applications

Two main challenges in robotics:

- execution time of the causal discovery analysis
- conduct the causal discovery analysis online



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Robotics Applications
F-PCMCI algorithm

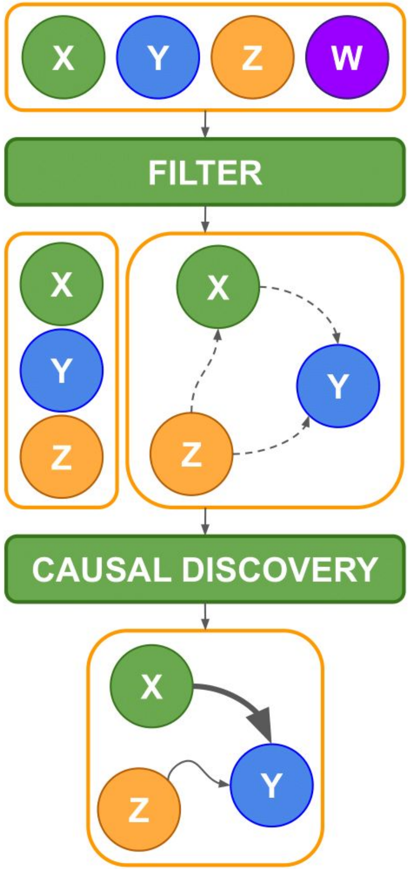
PCMCI computational cost depends on:

- the length of the time-series
- the number of variables

F-PCMCI steps:

- takes in input a prefixed set of variables
- the Transfer Entropy-based filter analyses and removes irrelevant variables (e.g., constants or isolated ones). The reduced variable set is used to create a hypothetical causal model
- the hypothetical causal model needs to be validated by a proper causal analysis, which is performed by PCMCI

This strategy enables **faster** and **more accurate** causal discovery

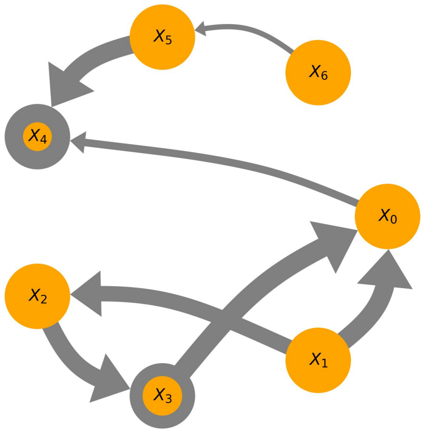


Castri, Luca, et al. "Enhancing Causal Discovery from Robot Sensor Data in Dynamic Scenarios." (2023).

Robotics Applications
F-PCMCI algorithm

$$
\begin{cases}
x_0(t) = 2x_1(t-1) + 3x_3(t-1) + \eta_0 \\
x_1(t) = \eta_1 \\
x_2(t) = 1.1x_1(t-1)^2 + \eta_2 \\
x_3(t) = x_3(t-1) \cdot x_2(t-1) + \eta_3 \\
x_4(t) = x_4(t-1) + x_5(t-1) \cdot x_0(t-1) + \eta_4 \\
x_5(t) = \eta_5 \\
x_6(t) = \eta_6
\end{cases}
$$

PCMCI



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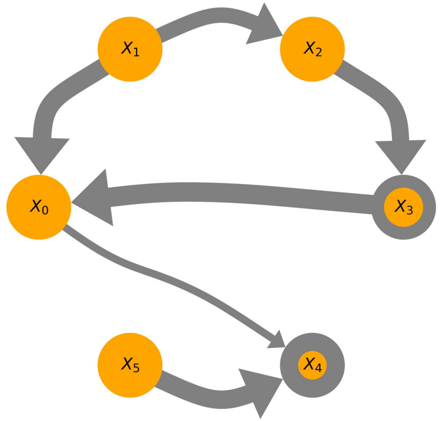

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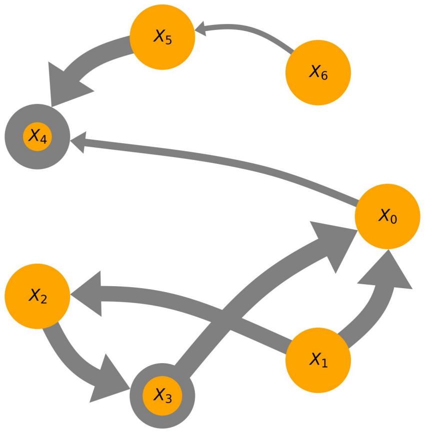
Isolated

PCMCI



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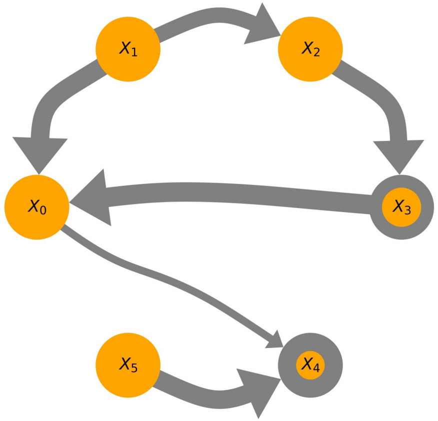

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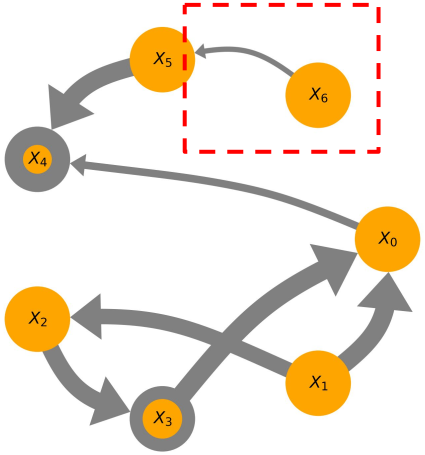
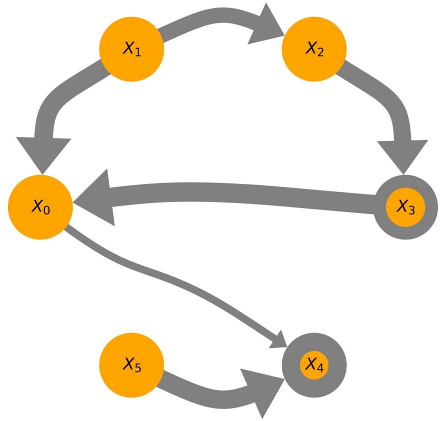
Isolated

PCMCI
~9mins



F-PCMCI
3mins



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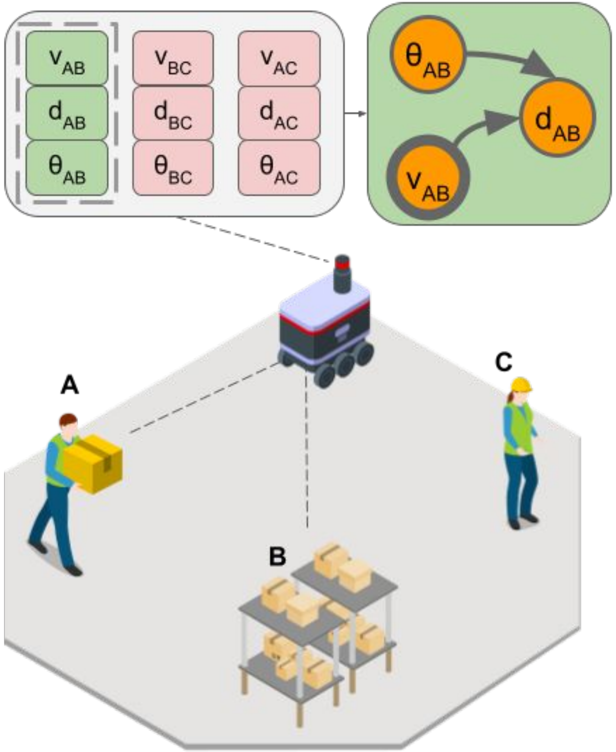
Robotics Applications
F-PCMCI algorithm

Considering the interaction scenario modelled by three variables

- v_{ij}: relative velocity between agent i and j
- d_{ij}: distance between agent i and j
- $theta_{ij}$: angle between agent i and j

Are all the observable variables useful to understand the observed scenario?

We need to filter the variables before conducting the causal analysis



Robotics Applications

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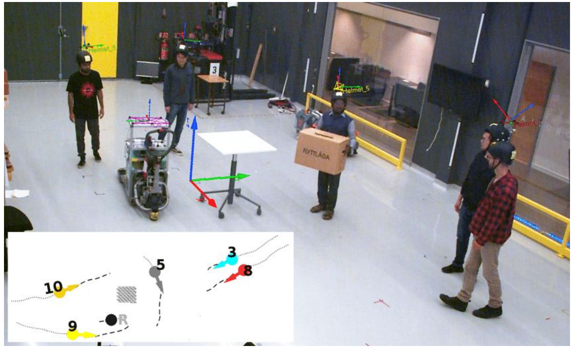

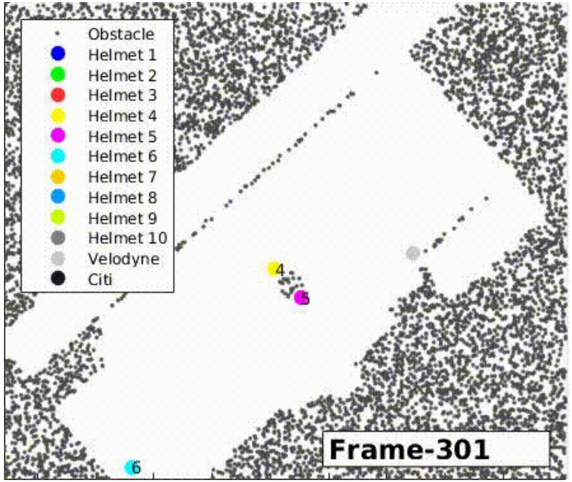

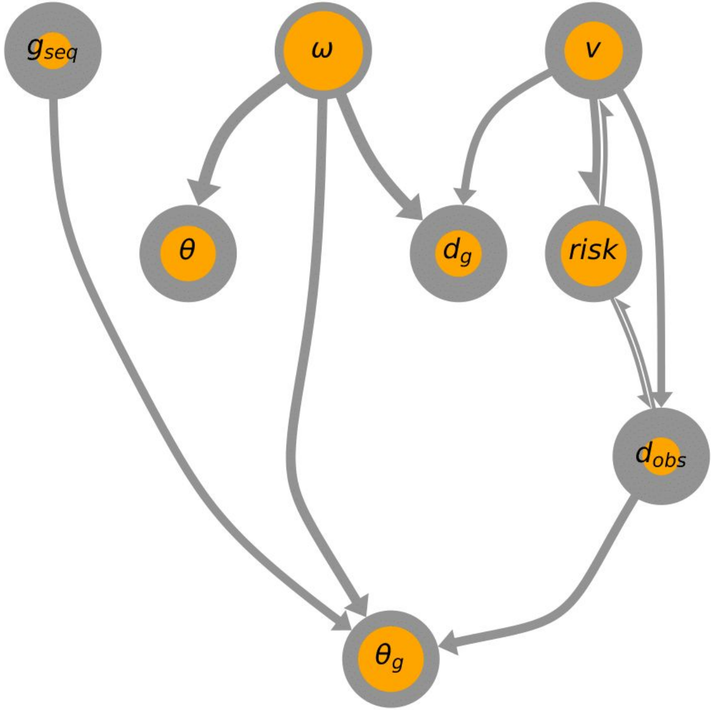

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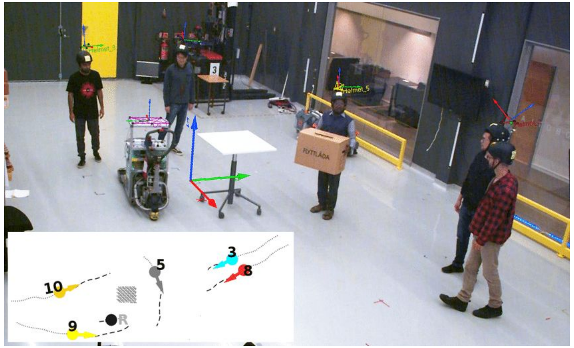
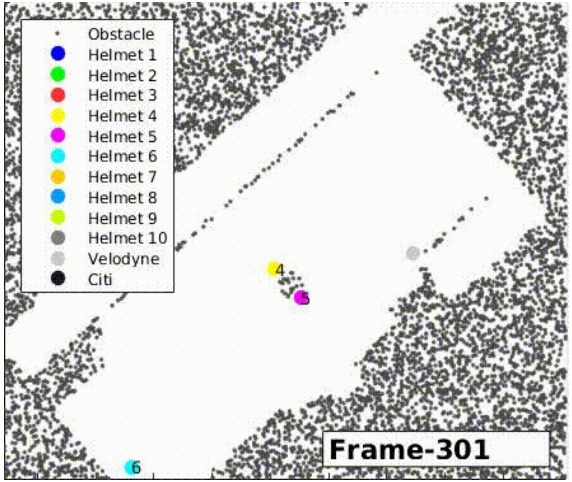
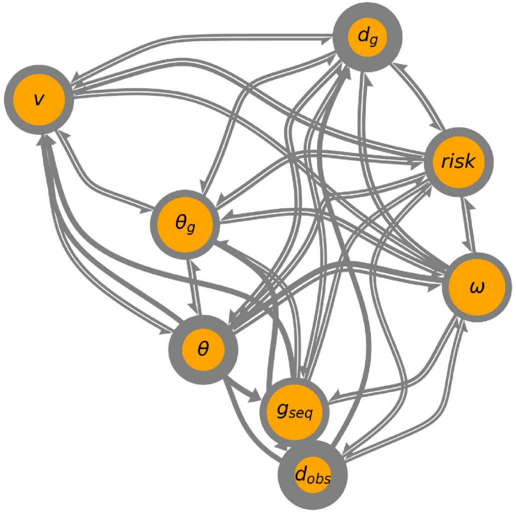
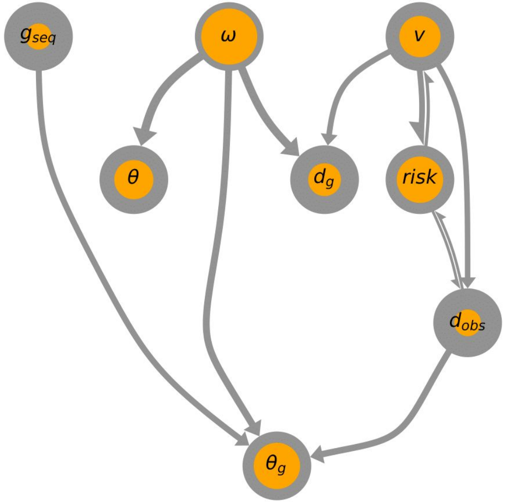






PCMCI ~80mins



F-PCMCI ~18mins

Castri, Luca, et al. "Enhancing Causal Discovery from Robot Sensor Data in Dynamic Scenarios." (2023).

Outline

- Causal Discovery for Time-series Data
  - PCMCI algorithm

- Robotics Applications
  - F-PCMCI algorithm
  - **ROS-Causal**

What is Robot Operating System (ROS)?

people tracker

navigation stack

...

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ROS

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navigation stack

...

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ROS

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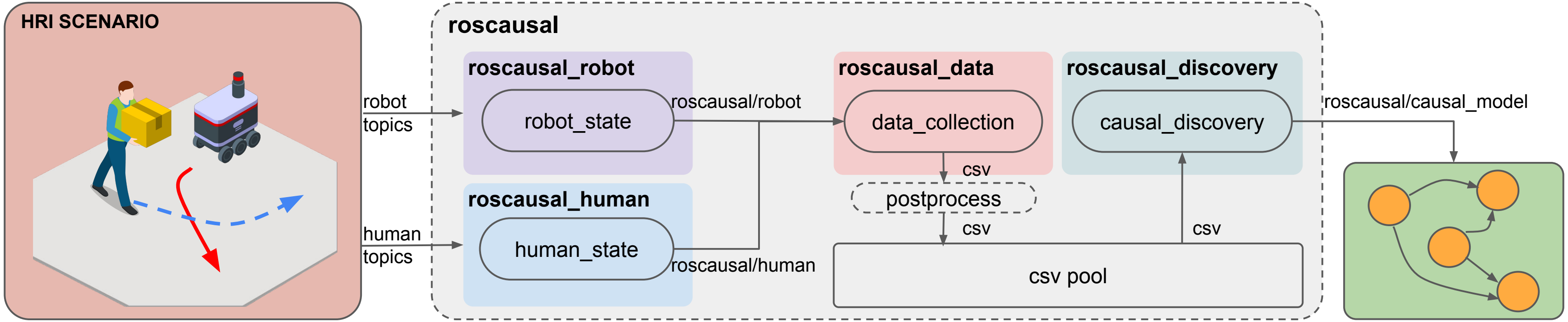
ROS-Causal

Robotics Applications

ROS-Causal

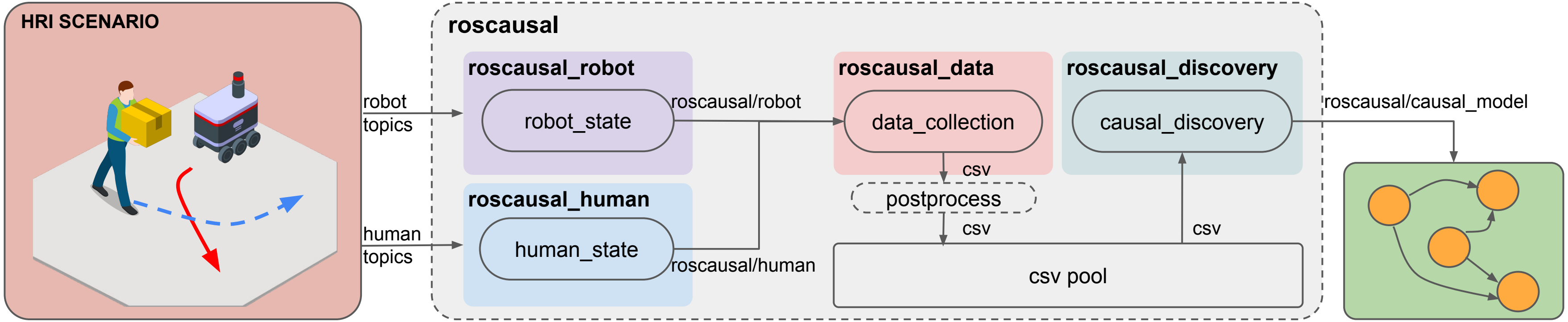


- ROS-Causal extracts and collects data from a HRI scenario, such as agents' trajectories, and performs causal analysis on the collected data in a batched manner. It is composed by four different rosnodes:
  - roscausal_robot
  - roscausal_human
  - roscausal_data
  - roscausal_discovery

Castri, Luca, et al. "ROS-Causal: A ROS-based Causal Analysis Framework for Human-Robot Interaction Applications" (2024).

Robotics Applications

ROS-Causal



roscausal_robot: collects data from several rostopics related to the robot (e.g., position, velocity, target position, etc.), and merge them into a single rostopic: *roscausal/robot*

roscausal_human: collects data from several rostopics related to the human (e.g., position, velocity, target position, etc.), and merge them into a single rostopic: *roscausal/human*

roscausal_data: subscribes to the topics */roscausal/robot* and */roscausal/human* and begins collecting data in a CSV file. Once the desired time-series length (rosparam) is reached, the node provides the option to post-process the data and finally saves the CSV file into a designated folder.

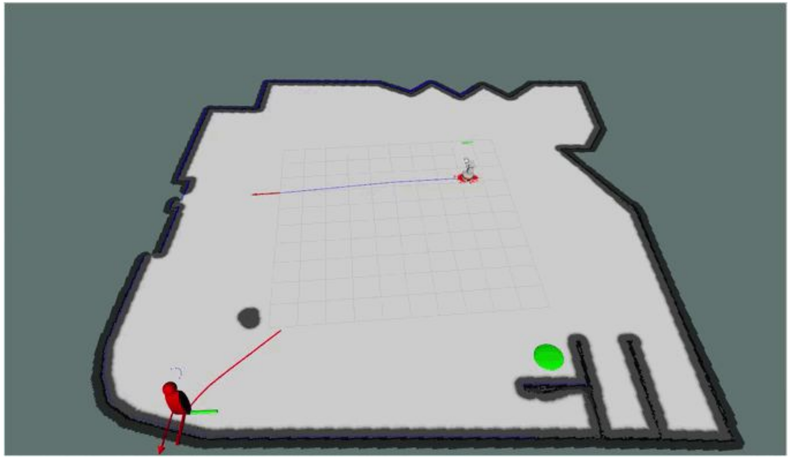
roscausal_discovery: performs causal discovery analysis on the collected data and publishes the result on the *roscausal/causal_model* rostopic. So far, it incorporates two causal discovery methods: PCMCI and F-PCMCI.

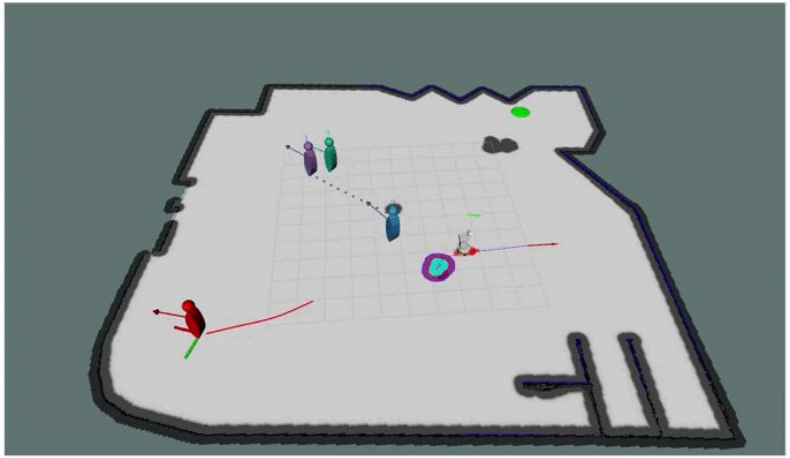
Castri, Luca, et al. "ROS-Causal: A ROS-based Causal Analysis Framework for Human-Robot Interaction Applications" (2024).

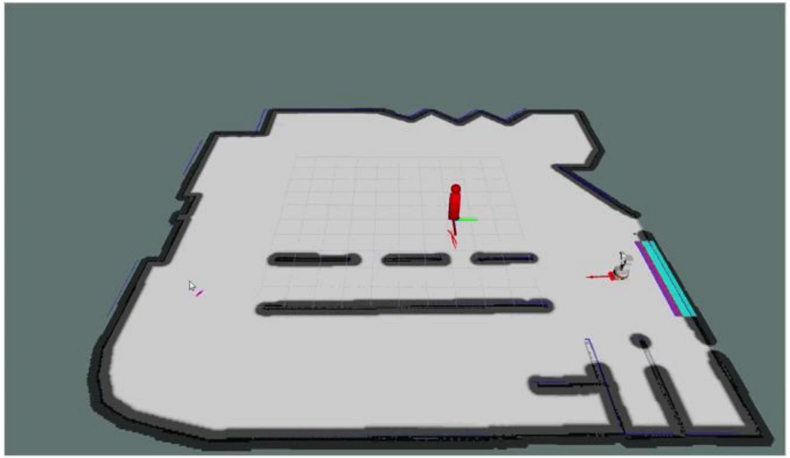
ROS-Causal_HRISim

HRI simulator involving:

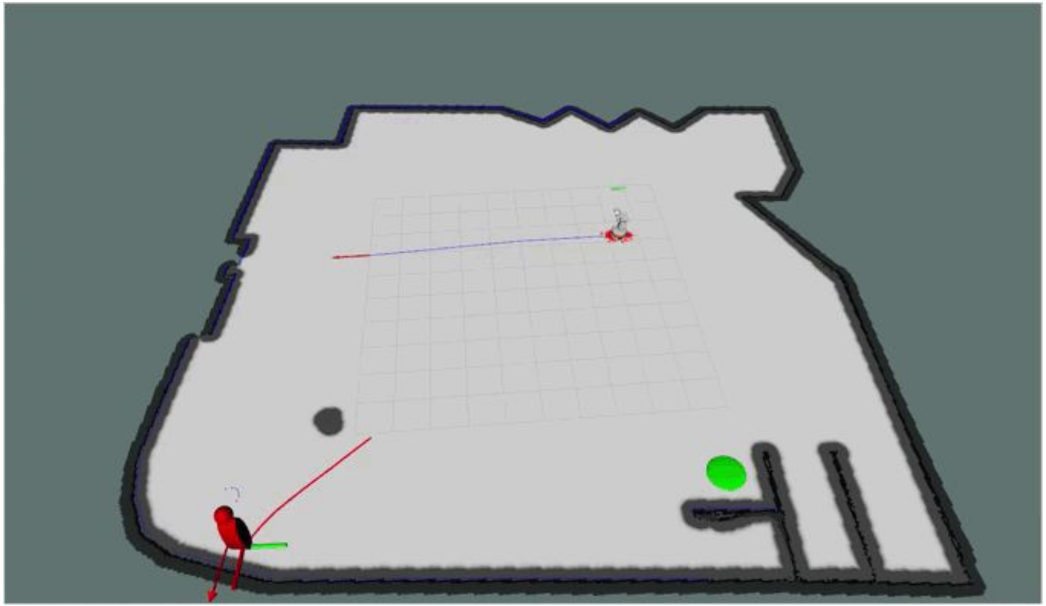
- TIAGo robot
- pedestrians

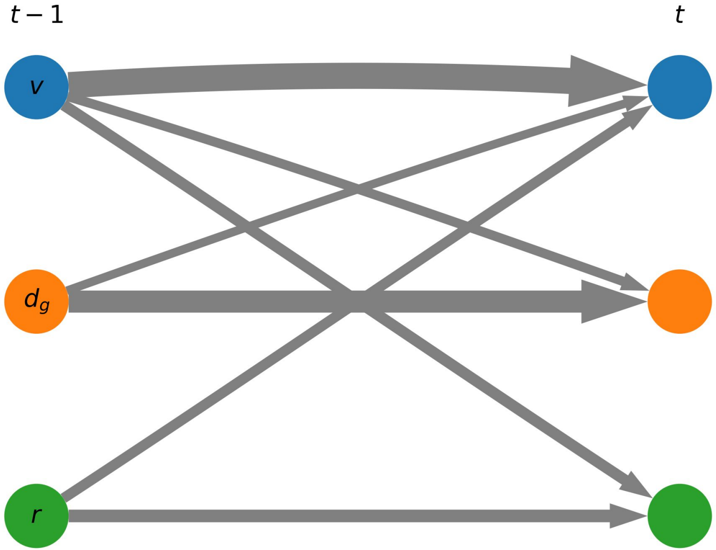




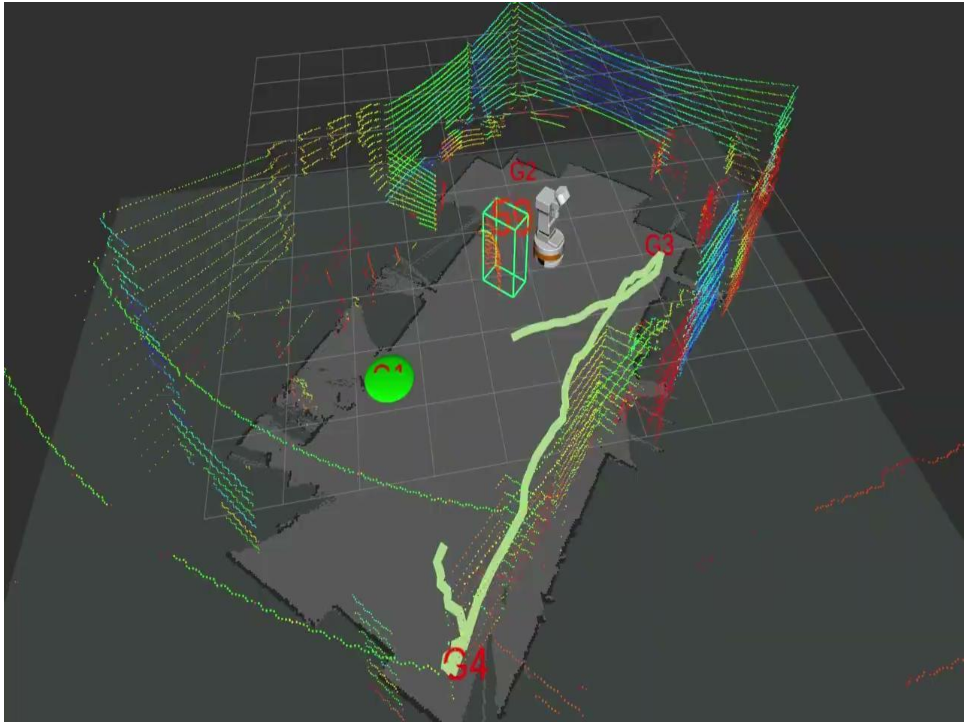


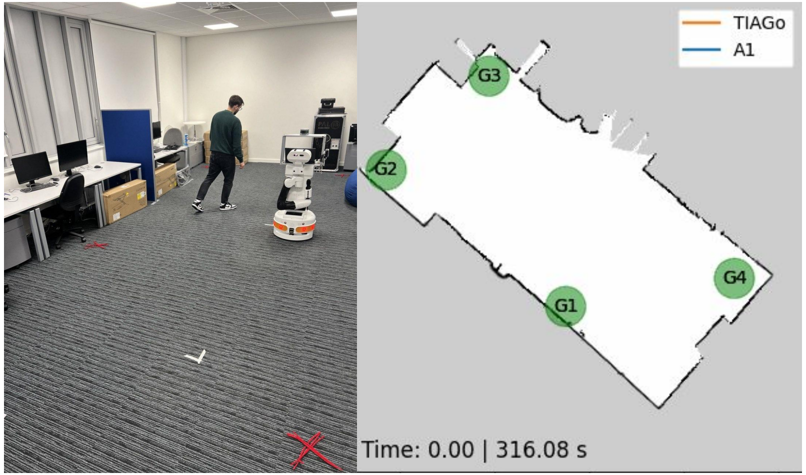
Robotics Applications
ROS-Causal

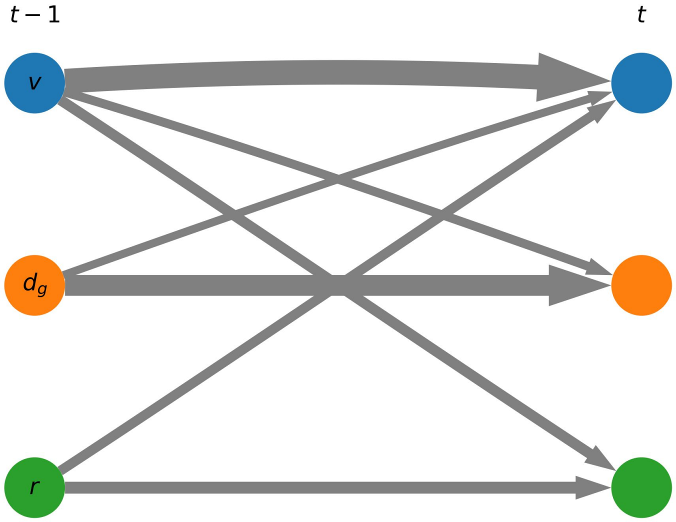




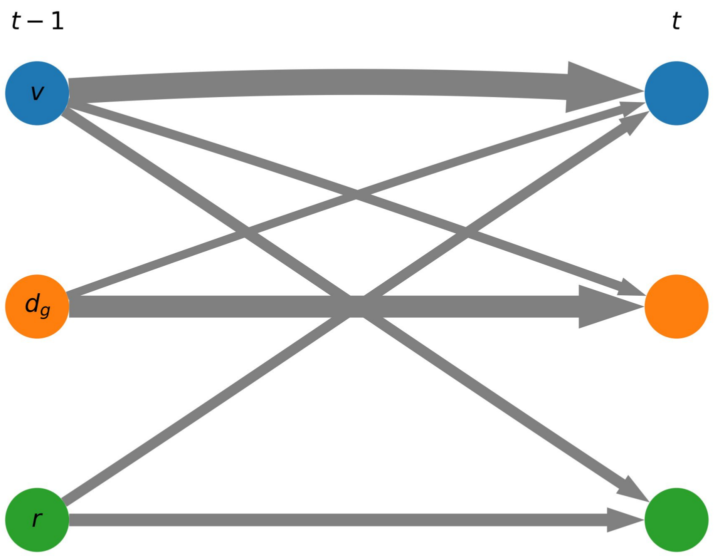
Robotics Applications
ROS-Causal





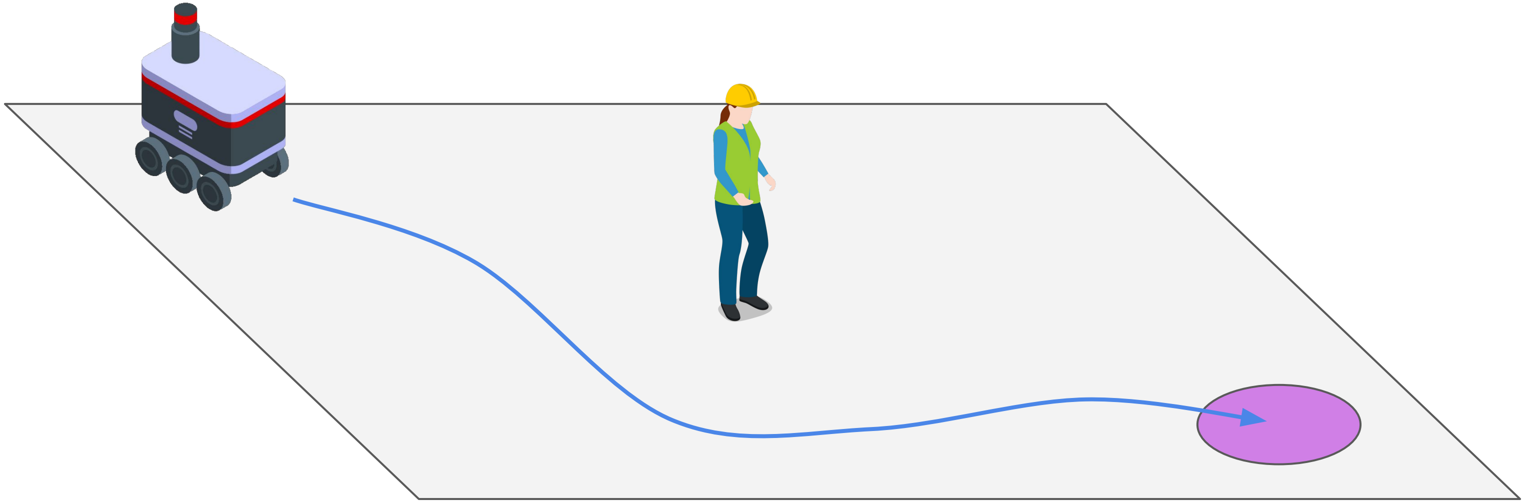


- What can a reconstructed causal model be useful for?
  - Prediction
  - Motion planning
  - Decision making



- What can a reconstructed causal model be useful for?
  - Prediction
  - Motion planning
  - Decision making

Trajectory generated by a planner not
accounting for human reaction to robot’s actions



Robotics Applications
ROS-Causal

- What can a reconstructed causal model be useful for?
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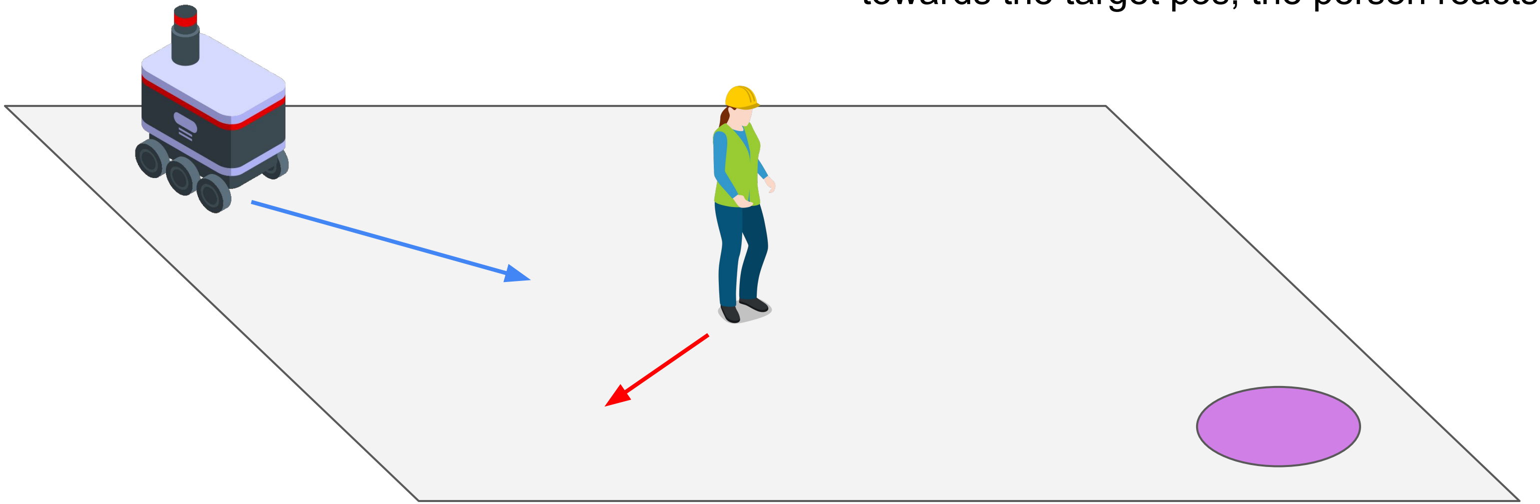
Trajectory generated by a planner not accounting for human reaction to robot's actions

But
What if, as soon as the robot starts moving towards the target pos, the person reacts?



Robotics Applications

ROS-Causal

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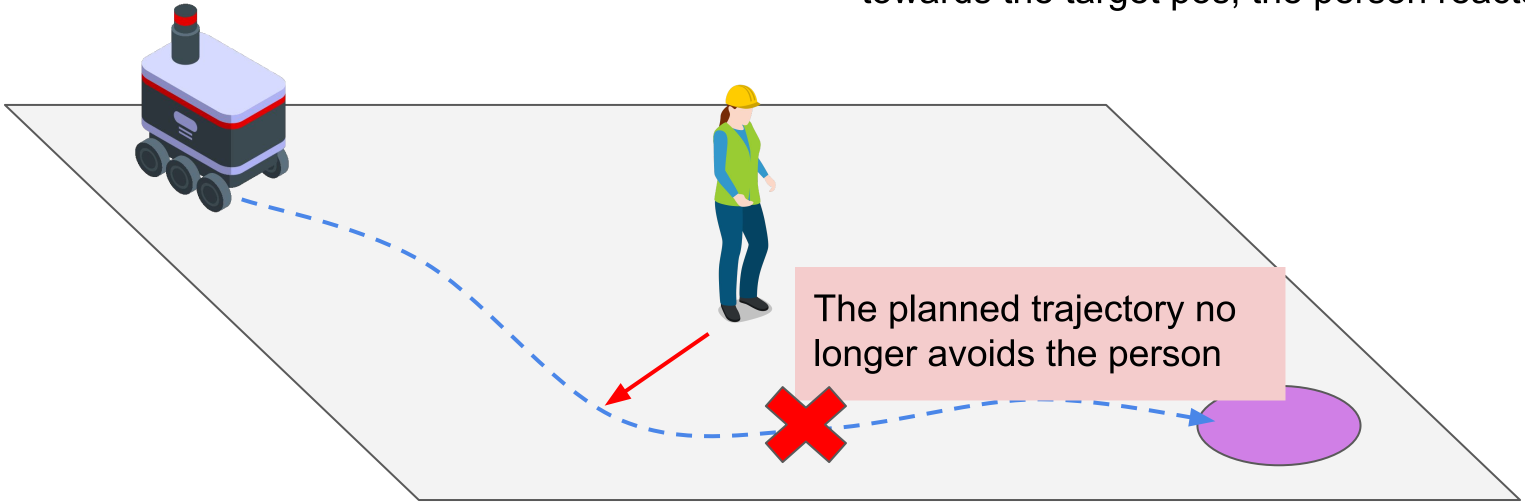

- What can a reconstructed causal model be useful for?
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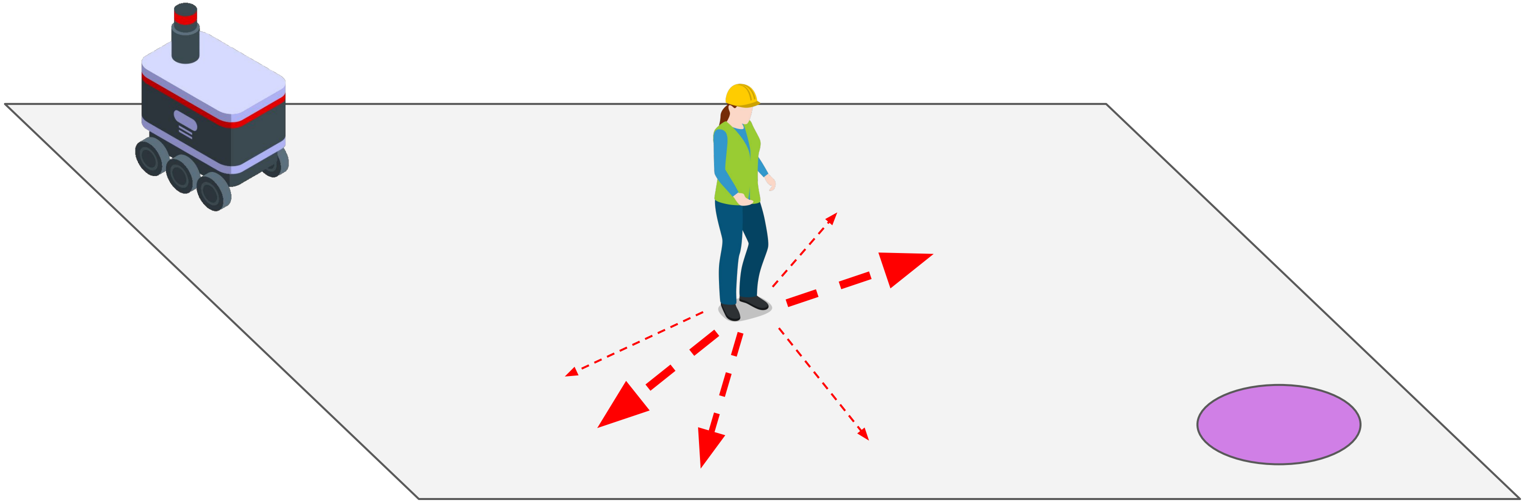
Trajectory generated by a planner not accounting for human reaction to robot's actions

Knowing the causal model

- can facilitate the prediction of the person spatial behaviours



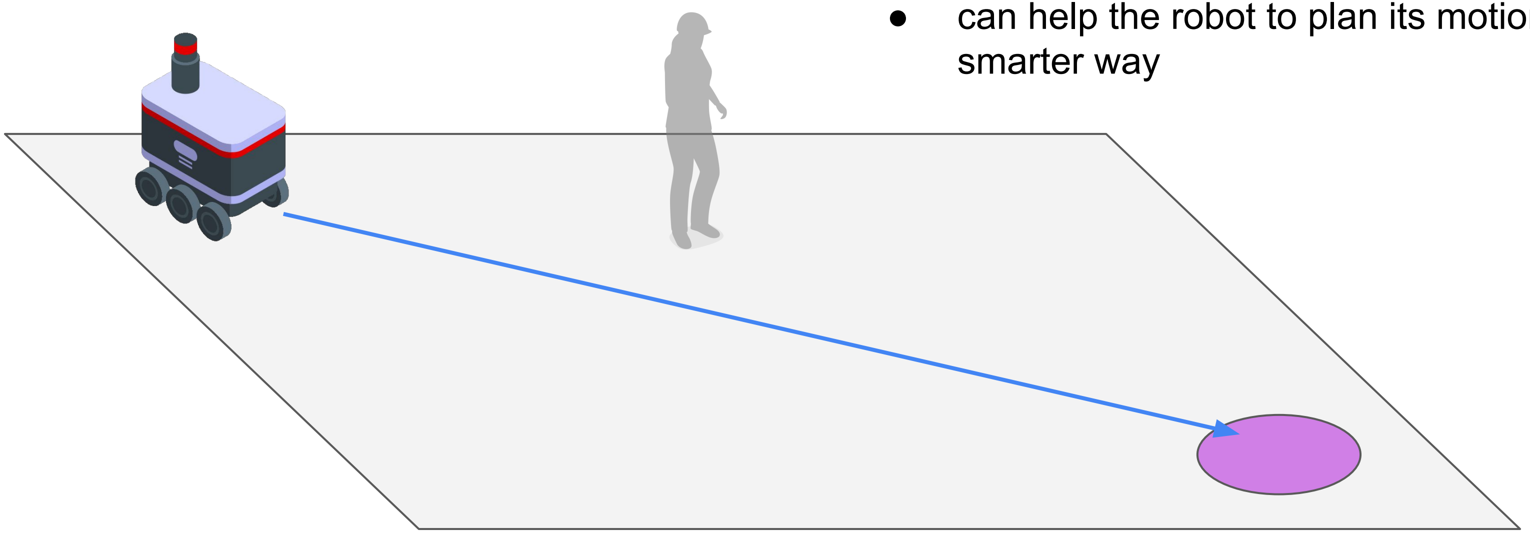
Robotics Applications
ROS-Causal

- What can a reconstructed causal model be useful for?
    - Prediction
    - Motion planning
    - Decision making

Trajectory generated by a planner not accounting for human reaction to robot's actions

Knowing the causal model

- can facilitate the prediction of the person spatial behaviours
- can help the robot to plan its motion in a smarter way



Reference

- Runge, J., Nowack, P., Kretschmer, M., Flaxman, S. and Sejdinovic, D., 2019. Detecting and quantifying causal associations in large nonlinear time series datasets. Science advances, 5(11).
- Castri, L., Mghames, S., Hanheide, M. and Bellotto, N., 2023. Enhancing Causal Discovery from Robot Sensor Data in Dynamic Scenarios. In 2nd Conference on Causal Learning and Reasoning.
- Castri, L., Beraldo, G., Mghames, S., Hanheide, M. and Bellotto, N., 2024. ROS-Causal: A ROS-based Causal Analysis Framework for Human-Robot Interaction Applications, Workshop on Causal Learning for Human-Robot Interaction (Causal-HRI), ACM/IEEE International Conference on Human-Robot Interaction (HRI).

PCMCI **F-PCMCI** **ROS-Causal**

Thank you, questions?