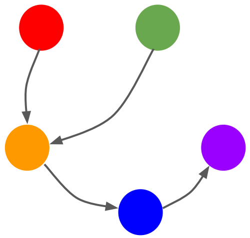
Causal Discovery



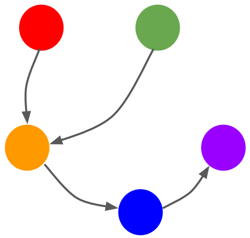
Luca Castri, PhD student @ University of Lincoln, UK

Outline

What is Causal Discovery?

Constraint-based methods
- Causal assumptions
- Markov Equivalence Class
- PC algorithm
- FCI algorithm

Causal Discovery with Time-series data
- PCMCI algorithm
- Robotics application



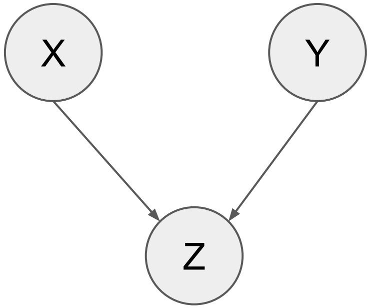
Causal Discovery

Causal Discovery: data→causal graph

	X	Y	Z
0	20.000000	100.0	340.000000
1	20.204082	100.0	340.408163
2	20.408163	100.0	340.816327
3	20.612245	100.0	341.224490
4	20.816327	100.0	341.632653
...	...	...	...
2495	29.183673	300.0	958.367347
2496	29.387755	300.0	958.775510
2497	29.591837	300.0	959.183673
2498	29.795918	300.0	959.591837
2499	30.000000	300.0	960.000000

2500 rows × 3 columns

→



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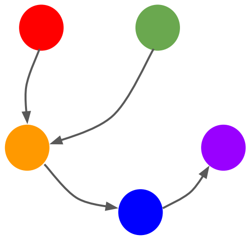

Constraint-based methods

They use conditional independence tests (constraints) to measure the independence between variables in order to identify the causal graph

PC - **P**eter Spirtes and **C**lark Glymour

FCI - **F**ast **C**ausal **I**nference

What are the assumptions?

- **Markov**
- **Faithfulness**
- **Causal Sufficiency**
- **Acyclicity**

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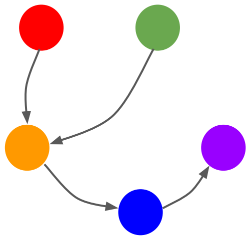

Constraint-based methods

Causal Assumptions

d-separation

Two (sets of) nodes X and Y are d-separated by a set of nodes Z if all the path between (any node in) X and (any node in) Y are blocked by Z

$$X \perp\!\!\!\perp_G Y | Z \quad \Rightarrow \quad X \perp\!\!\!\perp_P Y | Z$$

Markov assumption

d-separation in the graph implies conditional independencies in the distribution. **Causal graph ⇒ data**

Constraint-based methods

Causal Assumptions

d-separation

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Markov assumption

d-separation in the graph implies conditional independencies in the distribution. **Causal graph ⇒ data**

$$X \perp\!\!\!\perp_G Y | Z \quad \Leftarrow \quad X \perp\!\!\!\perp_P Y | Z$$

Faithfulness

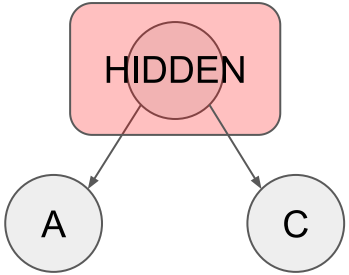
To do causal discovery we need to go the other way around.
Causal graph ⇐ data

It is the inverse of the Markov assumption and assesses:

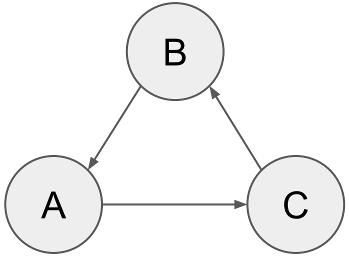
if two variables X and Y are independent conditioning on Z in the distribution then X and Y are d-separated by Z in the causal graph

Constraint-based methods
Causal Assumptions

Causal Sufficiency
There are no unobserved confounders of any of the variables in the graph



Acyclicity
No cycle in the graph



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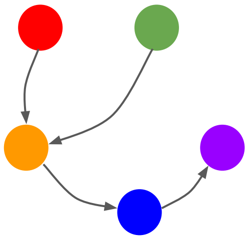
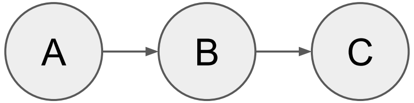


Constraint-based methods
Markov Equivalence Class

Chain



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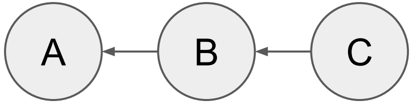
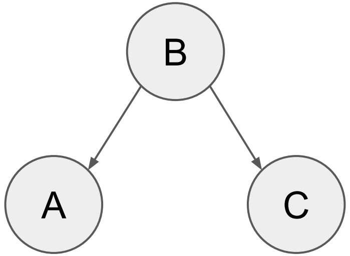


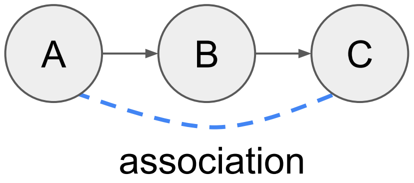
Fork



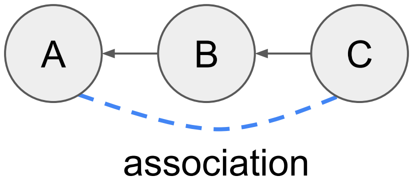
Constraint-based methods

Markov Equivalence Class

Chain



Chain



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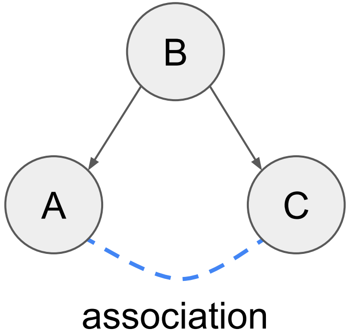
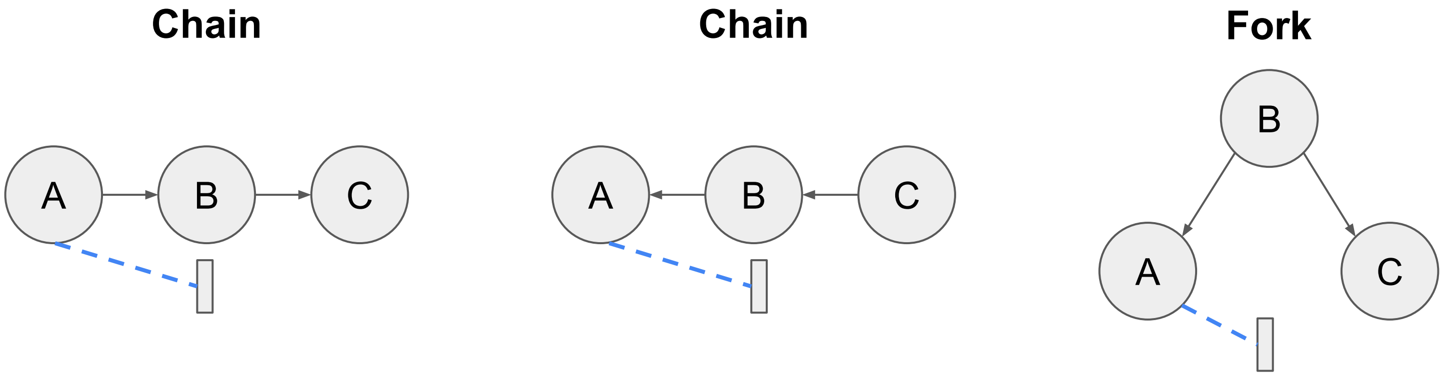


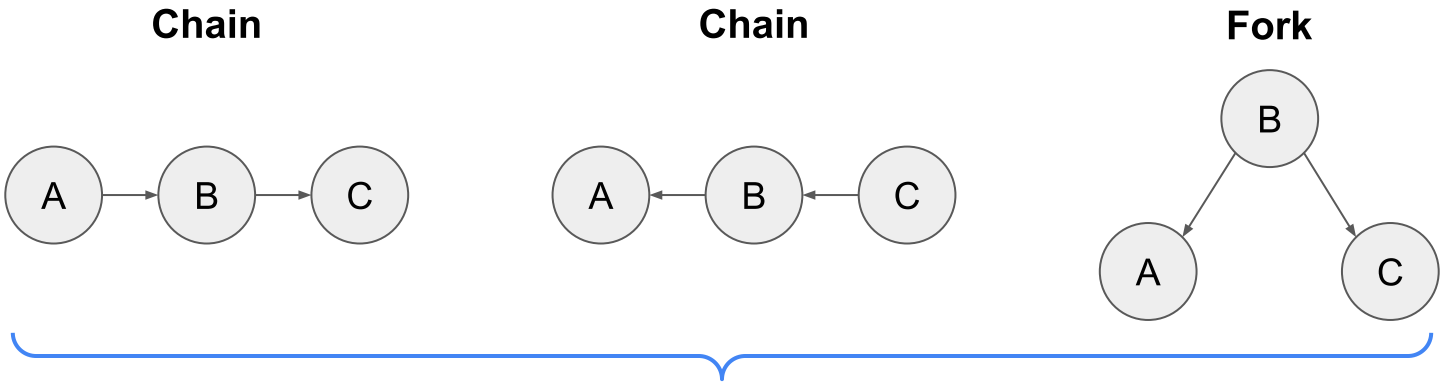
Constraint-based methods

Markov Equivalence Class



Markov: $A \perp\!\!\!\perp C | B$ They all imply the same conditional independence

Constraint-based methods
Markov Equivalence Class



Markov equivalent
They all belong to the same **Markov Equivalence Class**

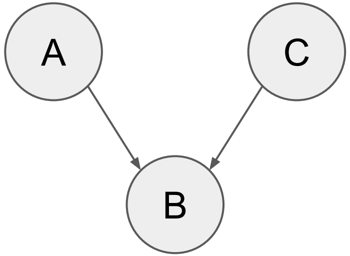
Markov: $A \perp\!\!\!\perp C | B$ They all imply the same conditional independence

Constraint-based methods

Markov Equivalence Class

Markov: $A \perp\!\!\!\perp C$

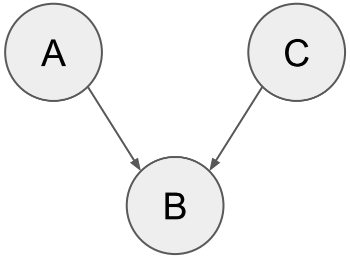
Collider



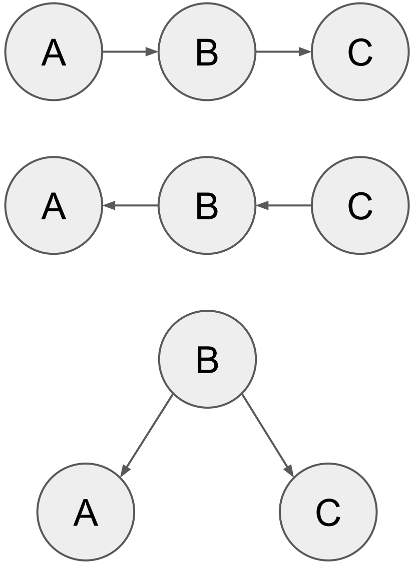
Constraint-based methods

Markov Equivalence Class

Markov: $A \perp\!\!\!\perp C$

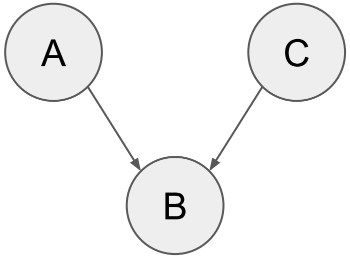


Markov: $A \perp\!\!\!\perp C \mid B$



Constraint-based methods
Markov Equivalence Class

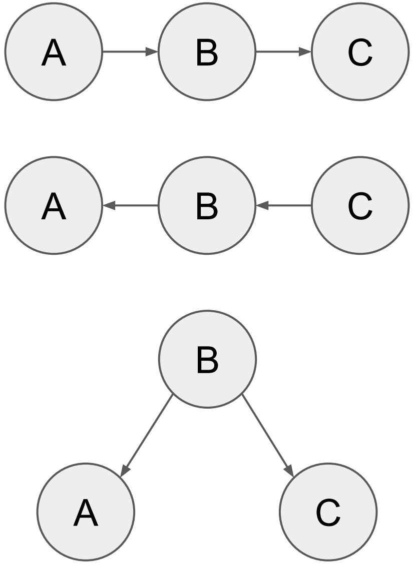
Markov: $A \perp\!\!\!\perp C$



If we find a triple of variables which satisfies this Markov assumption
⇒ collider configuration

COLLIDERS ARE IMPORTANT

Markov: $A \perp\!\!\!\perp C | B$



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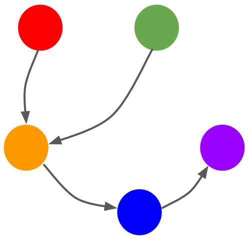

Constraint-based methods

PC algorithm

Markov | Faithfulness | Causal Sufficiency | Acyclicity

1. Start with fully connected and undirected graph
2. Identify the skeleton
3. Identify colliders and orient them
4. Orient the non-colliders edges (orientation propagation)

Constraint-based methods
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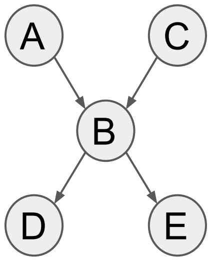

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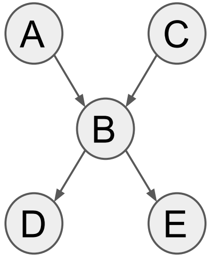

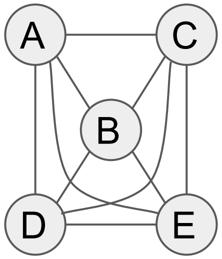

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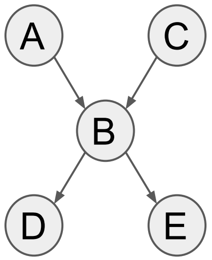

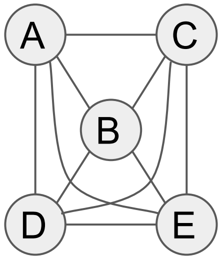

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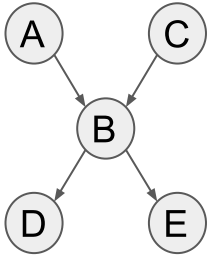
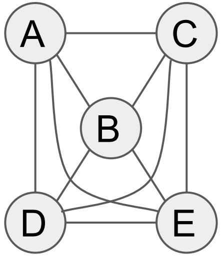
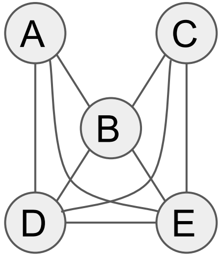


$A \perp\!\!\!\perp C | \{\}$





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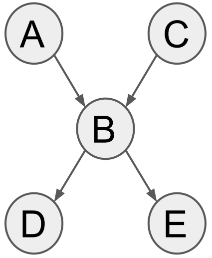

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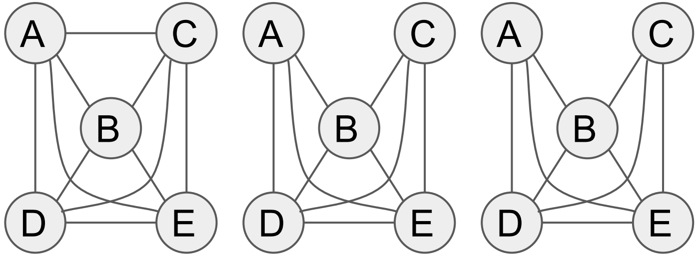

$A \perp\!\!\!\perp D | \{\}$ ✖
$D \perp\!\!\!\perp E | \{\}$ ✖
$C \perp\!\!\!\perp E | \{\}$ ✖
$A \perp\!\!\!\perp E | \{\}$ ✖
$C \perp\!\!\!\perp D | \{\}$ ✖

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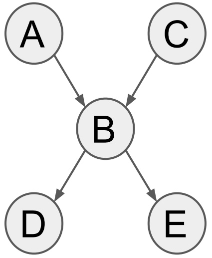
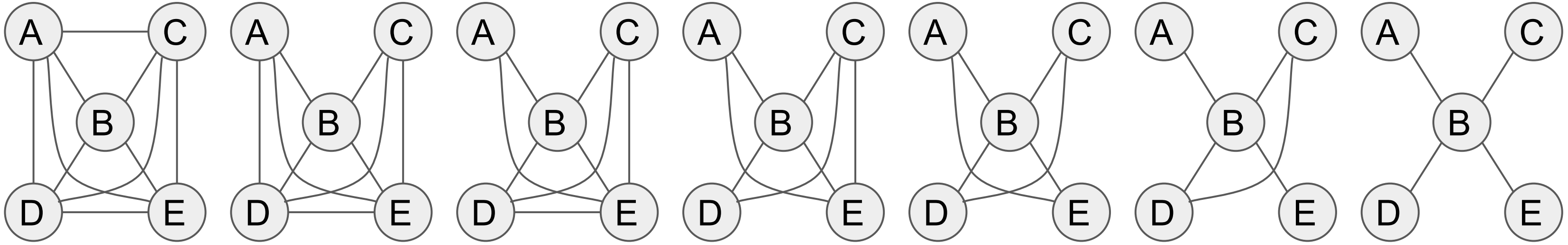


$A \perp\!\!\!\perp C|\{\}$ $A \perp\!\!\!\perp D|\{B\}$ $D \perp\!\!\!\perp E|\{B\}$ $C \perp\!\!\!\perp E|\{B\}$ $A \perp\!\!\!\perp E|\{B\}$ $C \perp\!\!\!\perp D|\{B\}$



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1. Start with fully connected and undirected graph
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   for any path X – Z – Y where there is no edge between
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   ⇒ X→Z←Y collider
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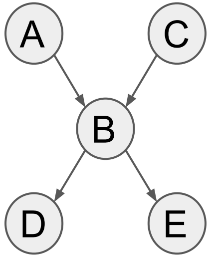

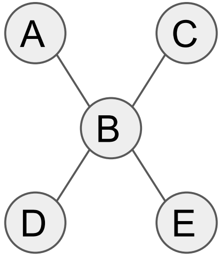

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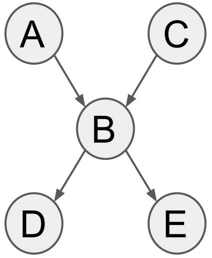
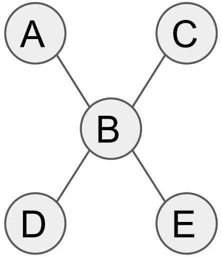
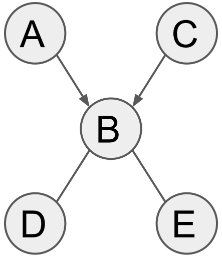


$A \perp\!\!\!\perp C | \{\}$ We removed A – C with conditioning set empty
⇒ unique markov equivalence class: **collider**
⇒ A→B←C





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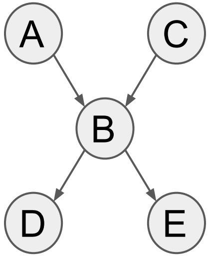
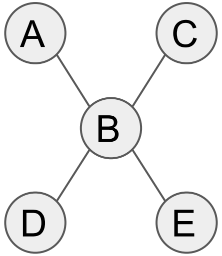
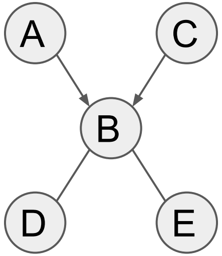






$A \perp\!\!\!\perp C | \{\}$

We removed A – C with conditioning set empty
⇒ unique markov equivalence class: **collider**
⇒ A→B←C

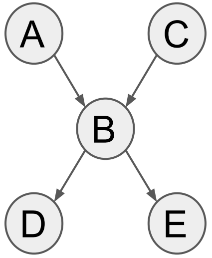
$A \perp\!\!\!\perp D | \{B\}$ $A \perp\!\!\!\perp E | \{B\}$
$D \perp\!\!\!\perp E | \{B\}$ $C \perp\!\!\!\perp D | \{B\}$
$C \perp\!\!\!\perp E | \{B\}$

We removed all these edges conditioning on B
⇒ markov equivalence class: **chain** or **fork**

Constraint-based methods

PC algorithm

Markov | Faithfulness | Causal Sufficiency | Acyclicity



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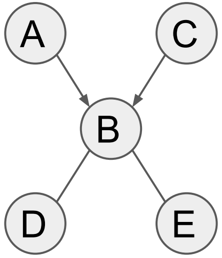
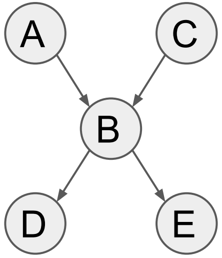
   any edge Z – Y part of a partially directed path X→Z – Y, where there is no edge between X and Y can be oriented as Z→Y



A→B – D, no edge between A and D ⇒ A→B→D

A→B – E, no edge between A and E ⇒ A→B→E



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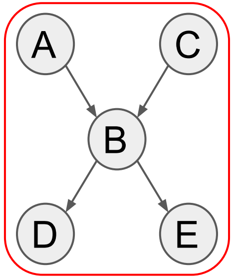

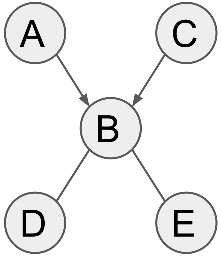

A→B – D, no edge between A and D ⇒ A→B→D
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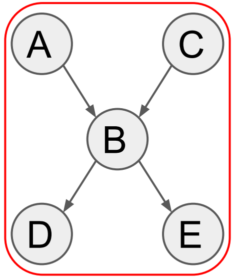

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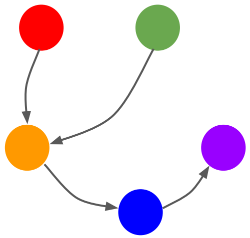
- PCMCI algorithm
- Robotics application

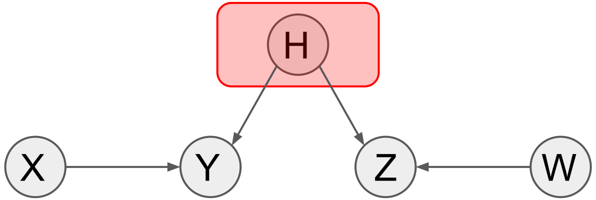


Constraint-based methods

FCI algorithm

Markov | Faithfulness | Causal Sufficiency | Acyclicity

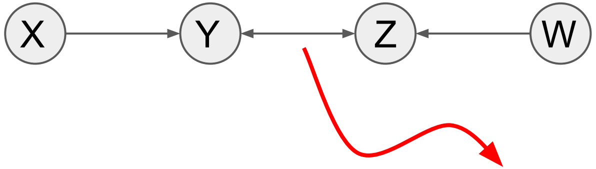
variation of the PC algorithm which tolerates hidden confounders.



skeleton

colliders identification

colliders identification



The bidirected edge indicates that there is at least one <u>hidden confounders</u>

Outline

What is Causal Discovery?

Constraint-based methods

- Causal assumptions
- Markov Equivalence Class
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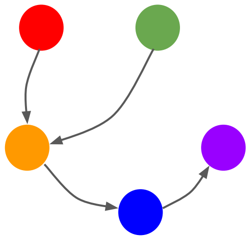
- FCI algorithm

Causal Discovery with Time-series data

- PCMCI algorithm
- Robotics application



Causal Discovery with Time-series data

The PC and FCI causal discovery method work well with discrete/categorical data.

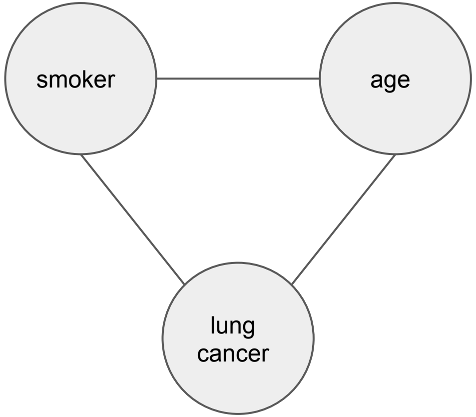
example

non-smoker **0** - smoker **1**

age under 50 **0** - age over 50 **1**

no lung cancer **0** - lung cancer **1**

Smoker	Age	Lung cancer
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



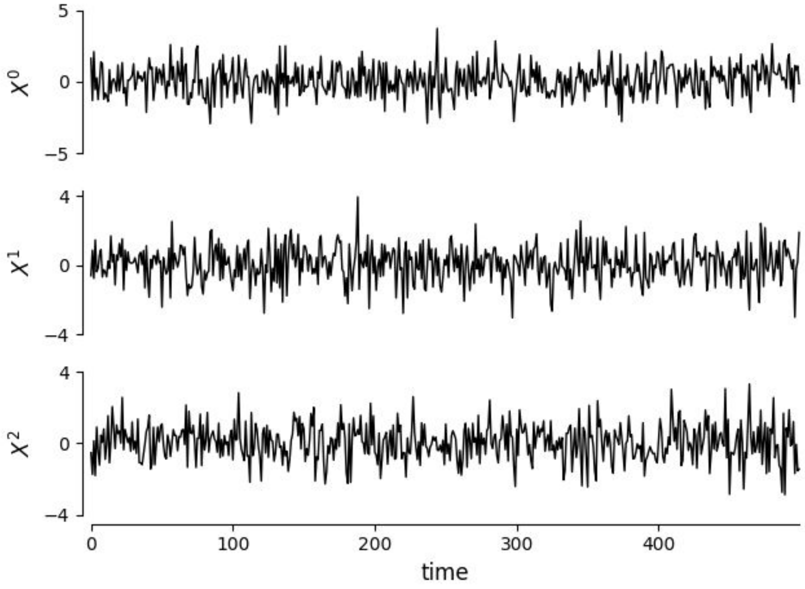
Causal Discovery with Time-series data

The PC and FCI causal discovery method work well with discrete/categorical data.

What if we deal with time-series data?
PC is inappropriate to use with time series data due to:

- time ordering
- lagged dependencies
- high false positive rates due to the autocorrelation.

$$\begin{cases} X_t^0 = 0.2(X_{t-1}^1)^2 + \eta_t^0 \\ X_t^1 = \eta_t^1 \\ X_t^2 = 0.3(X_{t-2}^1)^2 + \eta_t^2 \end{cases}$$



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Causal Discovery with Time-series data

- **PCMCI algorithm**
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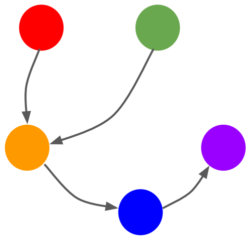

Causal Discovery with Time-series data

PCMCI algorithm

It consists of two main steps:

- **PC algorithm**
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- **MCI test**
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t | \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$

Key parameter: τ maximum time delay

Causal Discovery with Time-series data
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$$X_{t-\tau}^{i} \perp\!\!\!\perp X_{t}^{j} | \tilde{P}(X_{t-\tau}^{i}), \tilde{P}(X_{t}^{j})$$

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Causal Discovery with Time-series data

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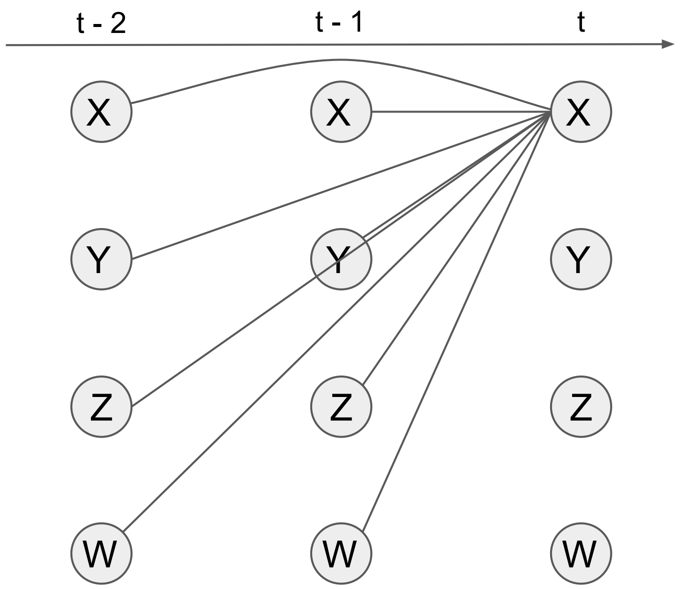

Causal Discovery with Time-series data
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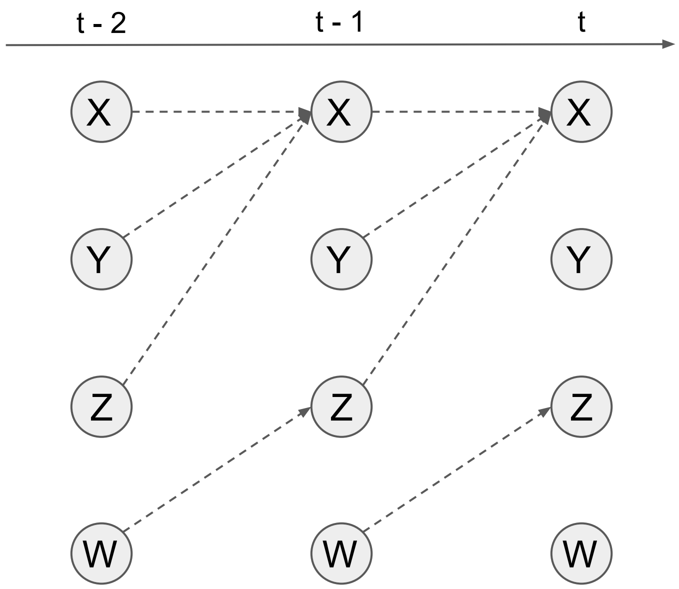

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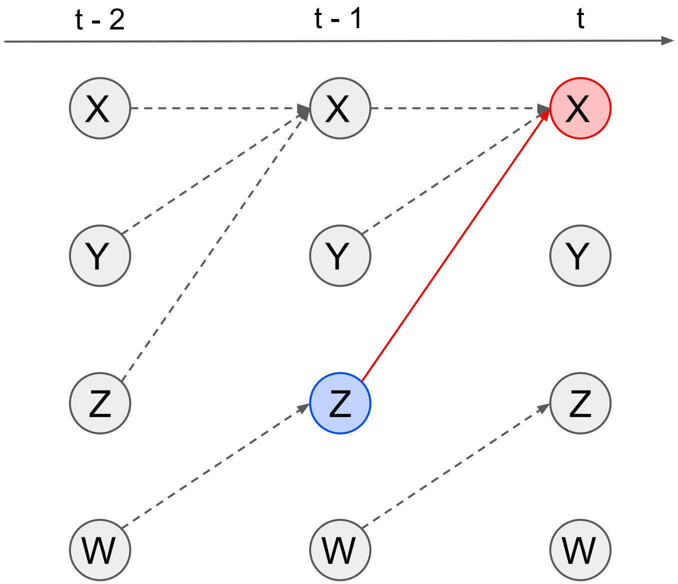

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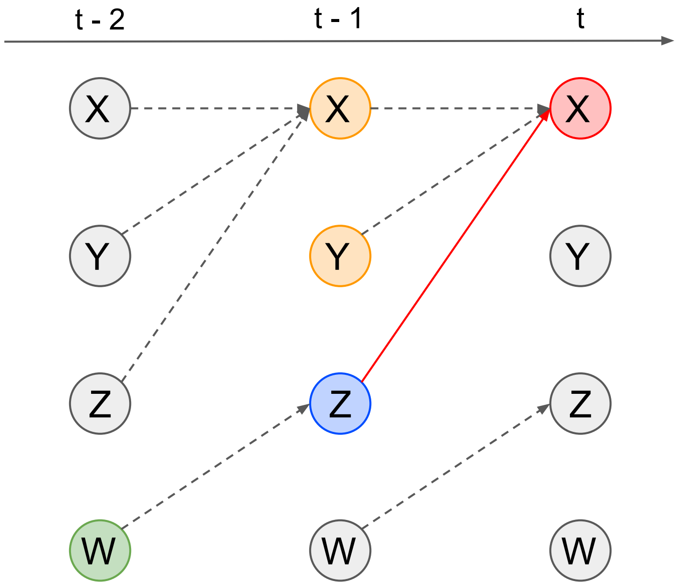

Causal Discovery with Time-series data

PCMCI algorithm

$$
\begin{cases}
X_t^0 = 0.4(X_{t-1}^1)^2 + \eta_t^0 \\
X_t^1 = \eta_t^1 \\
X_t^2 = 0.3(X_{t-2}^1)^2 + \eta_t^2
\end{cases}
$$

```
random_state = np.random.default_rng(seed=42)
data = random_state.standard_normal((500, 3))
for t in range(1, 500):
    data[t, 0] += 0.4*data[t-1, 1]**2
    data[t, 2] += 0.3*data[t-2, 1]**2
var_names = [r'$X^0$', r'$X^1$', r'$X^2$']

dataframe = pp.DataFrame(data, var_names=var_names)
```

```
gpdc = GPDC(significance='analytic', gp_params=None)
pcmci_gpdc = PCMCI(
    dataframe=dataframe,
    cond_ind_test=gpdc,
    verbosity=0)
```

```
results = pcmci_gpdc.run_pcmci(tau_max=2, pc_alpha=0.1, alpha_level = 0.01)
tp.plot_graph(
    val_matrix=results['val_matrix'],
    graph=results['graph'],
    var_names=var_names,
    show_colorbar=False,
    )
```

Causal Discovery with Time-series data

PCMCI algorithm

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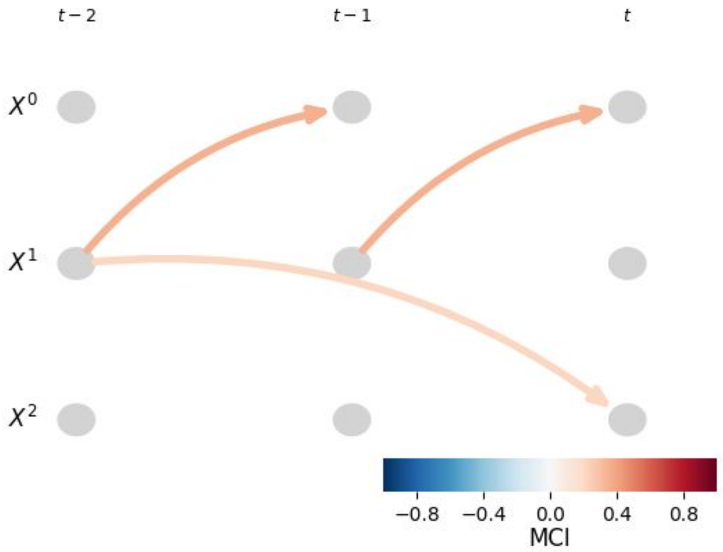

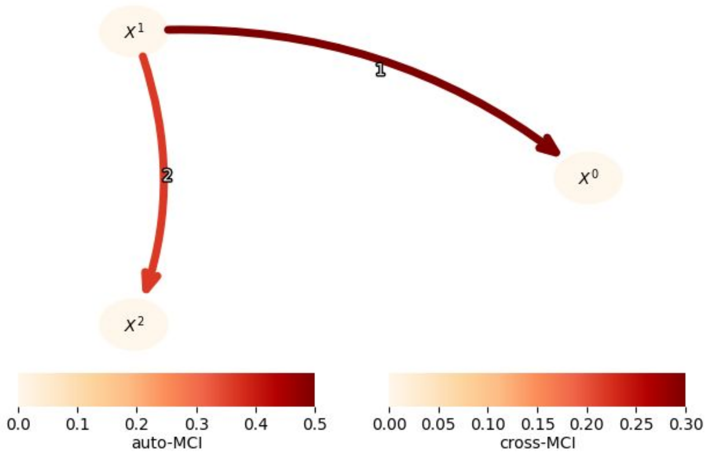

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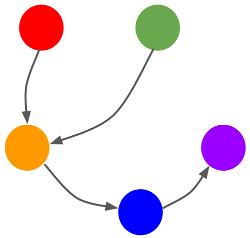
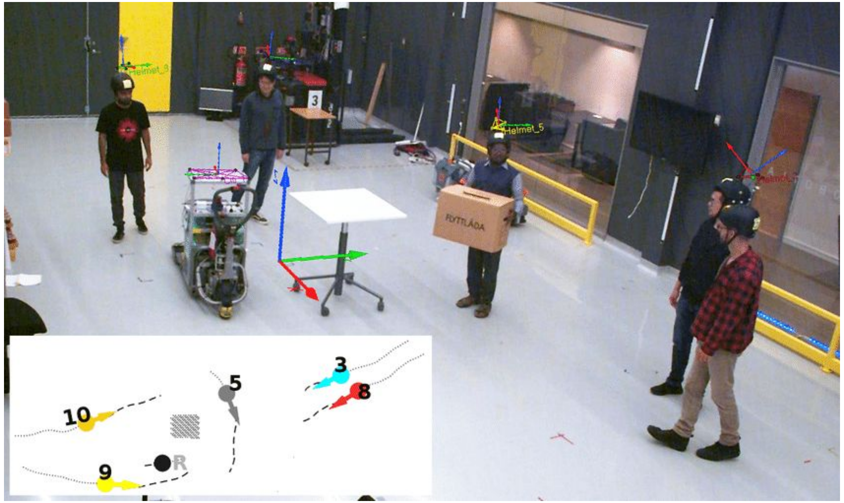
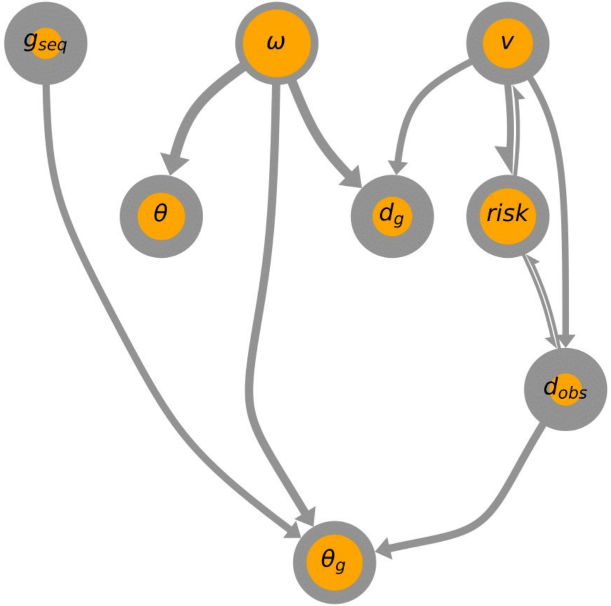


Causal Discovery with Time-series data

Robotics application

Only observational data





Castri, Luca, et al. "Causal Discovery of Dynamic Models for Predicting Human Spatial Interactions." (2022).
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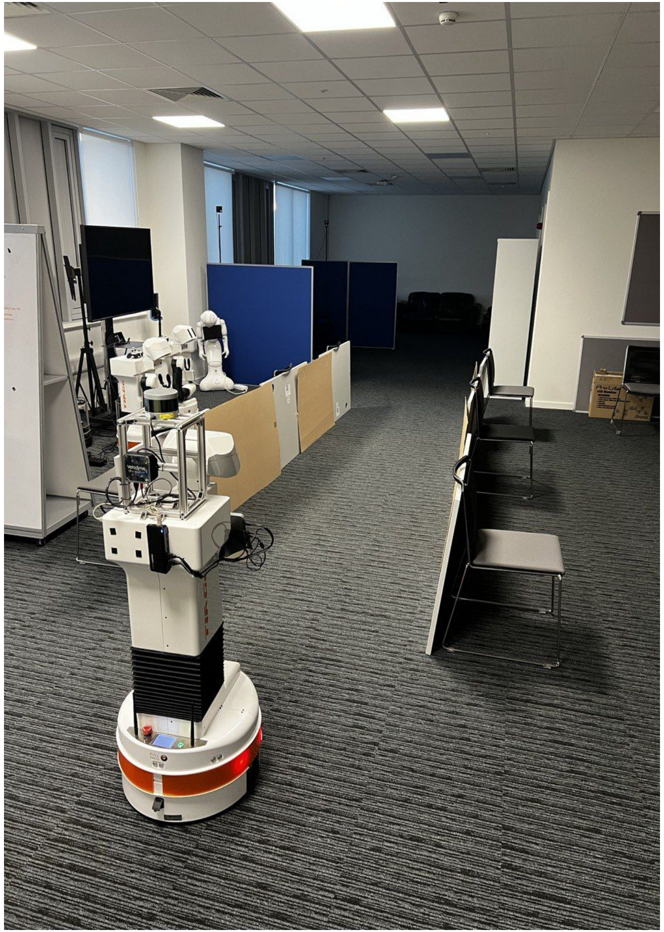
Causal Discovery with Time-series data
Robotics application

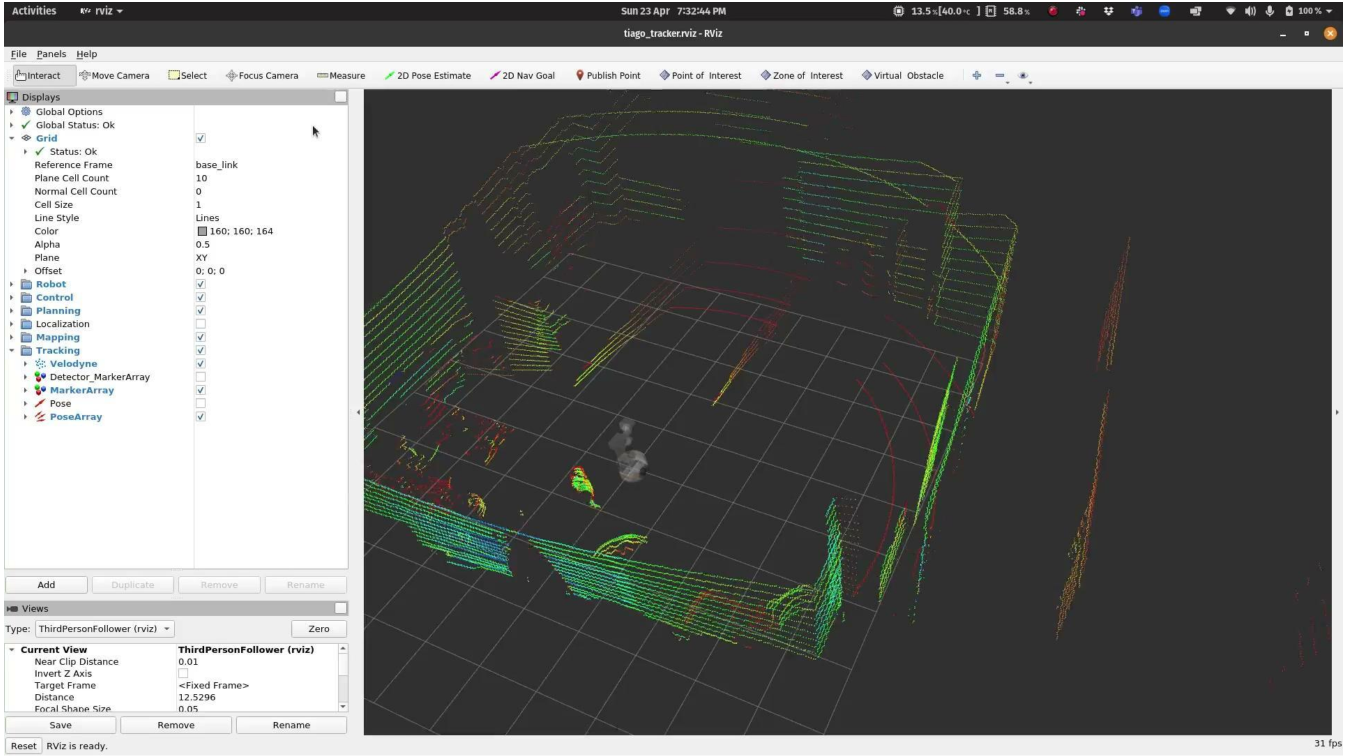
Observational and interventional data
The experiment consists of:

- TIAGo robot
- a human
- a “corridor”
- human overtakes the robot
  the robot tries to perform an action (intervention) to facilitate the overtake



Causal Discovery with Time-series data

Robotics application



CAUSAL MODEL

?

still ongoing..

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Thank you

questions?