Causal Discovery for Time-Series Data

Luca Castri, PhD student @ University of Lincoln, UK

Outline

- **Causal Discovery for Time-series Data**
  - PCMCI algorithm
- Robotics Applications
  - F-PCMCI algorithm
  - ROS-Causal

Causal Discovery for Time-series Data

The PC and FCI causal discovery method work well with discrete/categorical data.

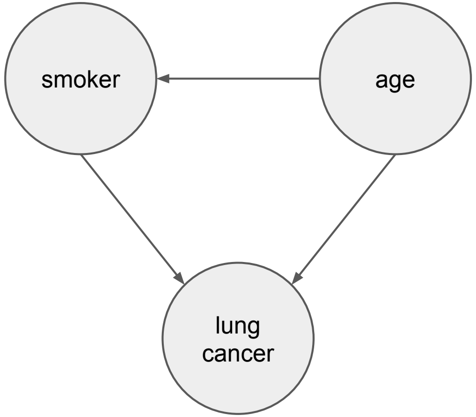
example

non-smoker **0** - smoker **1**

age under 50 **0** - age over 50 **1**

no lung cancer **0** - lung cancer **1**

Smoker	Age	Lung cancer
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



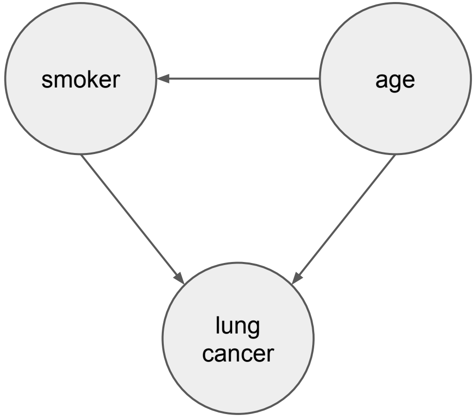
Causal Discovery for Time-series Data

The PC and FCI causal discovery method work well with discrete/categorical data.

example

non-smoker **0** - smoker **1**
age under 50 **0** - age over 50 **1**
no lung cancer **0** - lung cancer **1**

Smoker	Age	Lung cancer
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



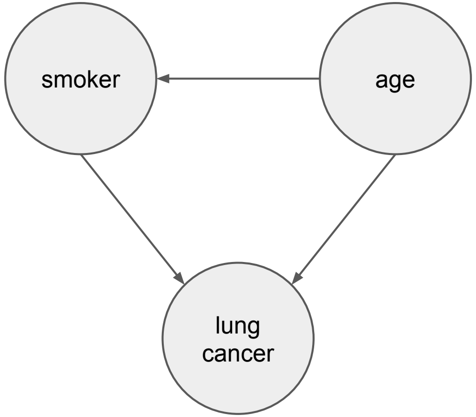
Causal Discovery for Time-series Data

The PC and FCI causal discovery method work well with discrete/categorical data.

example

non-smoker **0** - smoker **1**
age under 50 **0** - age over 50 **1**
no lung cancer **0** - lung cancer **1**

Smoker	Age	Lung cancer
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



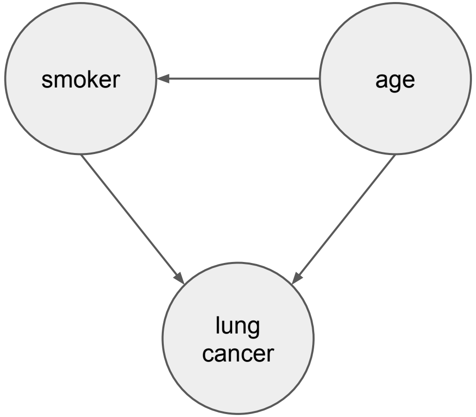
Causal Discovery for Time-series Data

The PC and FCI causal discovery method work well with discrete/categorical data.

example

non-smoker **0** - smoker **1**
age under 50 **0** - age over 50 **1**
no lung cancer **0** - lung cancer **1**

Smoker	Age	Lung cancer
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1



Causal Discovery for Time-series Data

The PC and FCI causal discovery method work well with discrete/categorical data.

example

non-smoker **0** - smoker **1**

age under 50 **0** - age over 50 **1**

no lung cancer **0** - lung cancer **1**

The order is not important

Smoker	Age	Lung cancer
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

=

Smoker	Age	Lung cancer
1	1	0
1	1	1
1	0	0
1	0	1
0	1	0
0	1	1
0	0	0
0	0	1

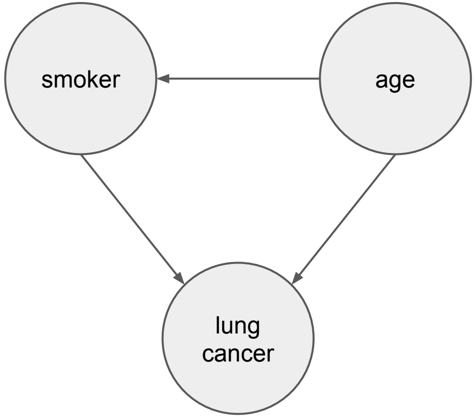
Causal Discovery for Time-series Data

The PC and FCI causal discovery method work well with discrete/categorical data.

example
non-smoker **0** - smoker **1**
age under 50 **0** - age over 50 **1**
no lung cancer **0** - lung cancer **1**

Time	Smoker	Age	Lung cancer
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

What if our data is time-dependent?



Causal Discovery for Time-series Data

The PC and FCI causal discovery method work well with discrete/categorical data.

example

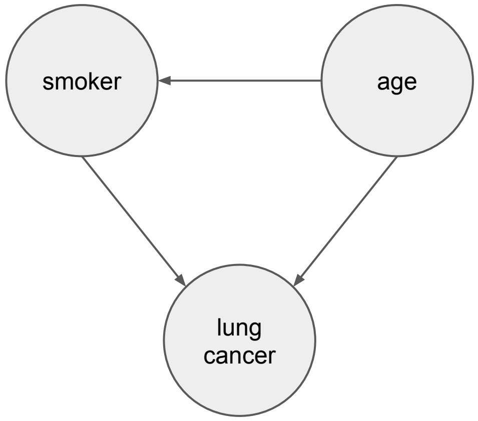
non-smoker **0** - smoker **1**

age under 50 **0** - age over 50 **1**

no lung cancer **0** - lung cancer **1**

Time	Smoker	Age	Lung cancer
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

What if our data is time-dependent?



We cannot only consider the contemporaneous relationships

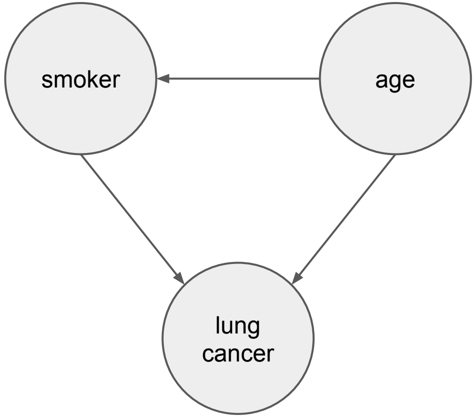
Causal Discovery for Time-series Data

The PC and FCI causal discovery method work well with discrete/categorical data.

example
non-smoker **0** - smoker **1**
age under 50 **0** - age over 50 **1**
no lung cancer **0** - lung cancer **1**

What if our data is time-dependent?

Time	Smoker	Age	Lung cancer
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1



We cannot only consider the contemporaneous relationships

Causal Discovery for Time-series Data

The PC and FCI causal discovery method work well with discrete/categorical data.

example

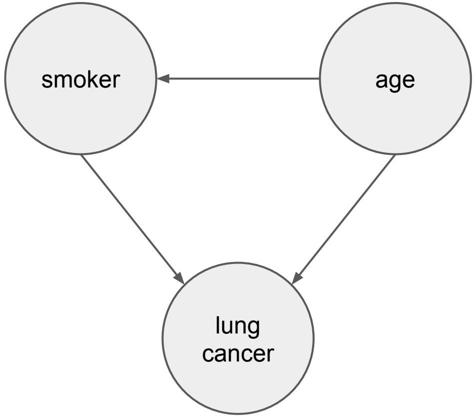
non-smoker **0** - smoker **1**

age under 50 **0** - age over 50 **1**

no lung cancer **0** - lung cancer **1**

What if our data is time-dependent?

Time	Smoker	Age	Lung cancer
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1



We cannot only consider the contemporaneous relationships

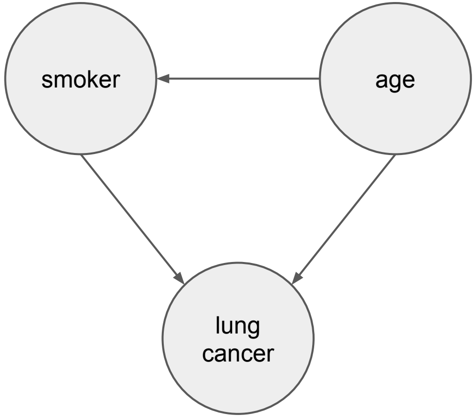
Causal Discovery for Time-series Data

The PC and FCI causal discovery method work well with discrete/categorical data.

example
non-smoker **0** - smoker **1**
age under 50 **0** - age over 50 **1**
no lung cancer **0** - lung cancer **1**

What if our data is time-dependent?

Time	Smoker	Age	Lung cancer
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1



We cannot only consider the contemporaneous relationships

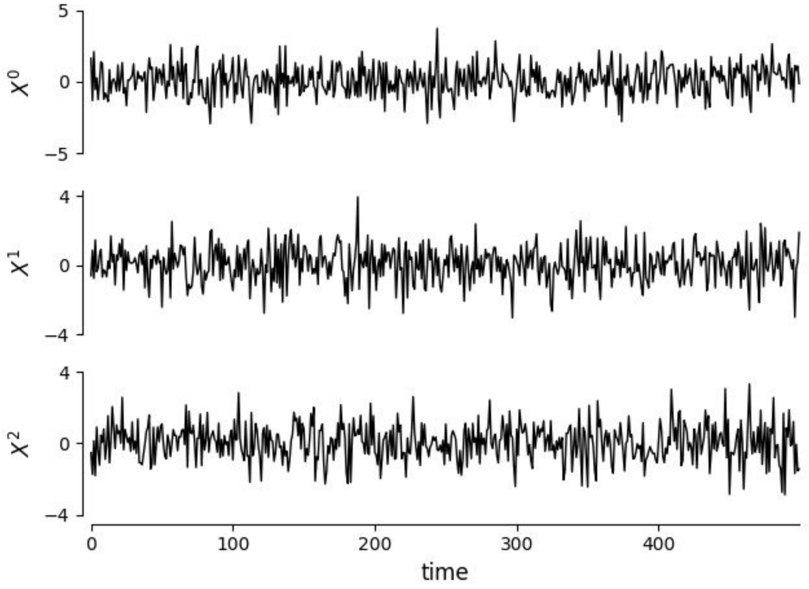
Causal Discovery for Time-series Data

The PC and FCI causal discovery method work well with discrete/categorical data.

What if we deal with time-series data?
PC/FCI is inappropriate to use with time series data due to:

- time ordering
- lagged dependencies
- high false positive rates due to the autocorrelation.

$$\begin{cases} X_t^0 = 0.2(X_{t-1}^1)^2 + \eta_t^0 \\ X_t^1 = \eta_t^1 \\ X_t^2 = 0.3(X_{t-2}^1)^2 + \eta_t^2 \end{cases}$$



Outline

- Causal Discovery for Time-series Data
  - **PCMCI algorithm**

- Robotics Applications
  - F-PCMCI algorithm
  - ROS-Causal

Causal Discovery for Time-series Data

PCMCI algorithm

It consists of two main steps:

- **PC algorithm**
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- **MCI test**
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t | \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$

Key parameter: τ maximum time delay

Causal Discovery for Time-series Data
PCMCI algorithm

It consists of two main steps:

- **PC algorithm**
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- **MCI test**
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t \mid \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$

Key parameter: τ maximum time delay

Causal Discovery for Time-series Data
PCMCI algorithm

It consists of two main steps:

- **PC algorithm**
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- **MCI test**
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t \,|\, \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$

Key parameter: τ maximum time delay

Causal Discovery for Time-series Data

PCMCI algorithm

It consists of two main steps:

- **PC algorithm**
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- **MCI test**
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t \mid \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$

Key parameter: τ maximum time delay

Causal Discovery for Time-series Data

PCMCI algorithm

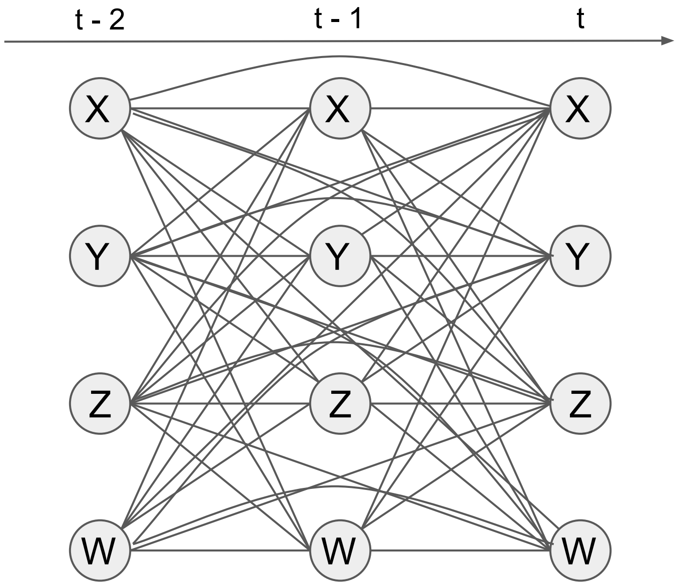
It consists of two main steps:

- **PC algorithm**
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- **MCI test**
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t \,|\, \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$

Key parameter: τ maximum time delay



Causal Discovery for Time-series Data

PCMCI algorithm

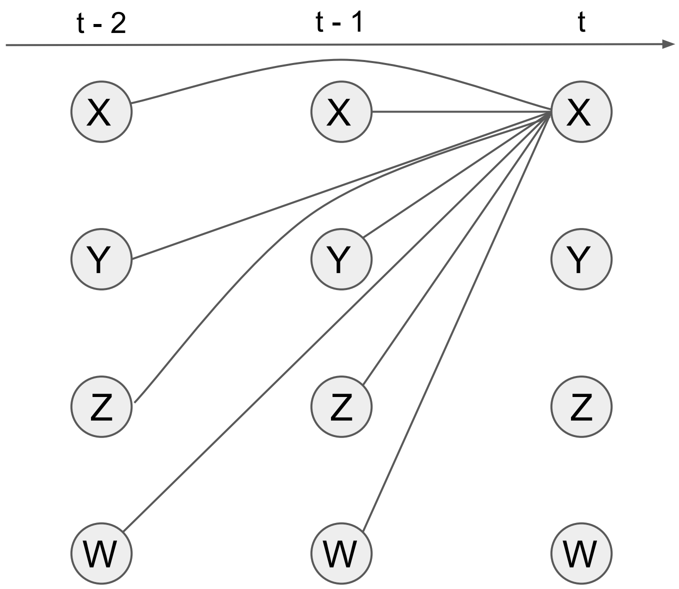
It consists of two main steps:

- **PC algorithm**
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- **MCI test**
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t | \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$

Key parameter: τ maximum time delay



Causal Discovery for Time-series Data

PCMCI algorithm

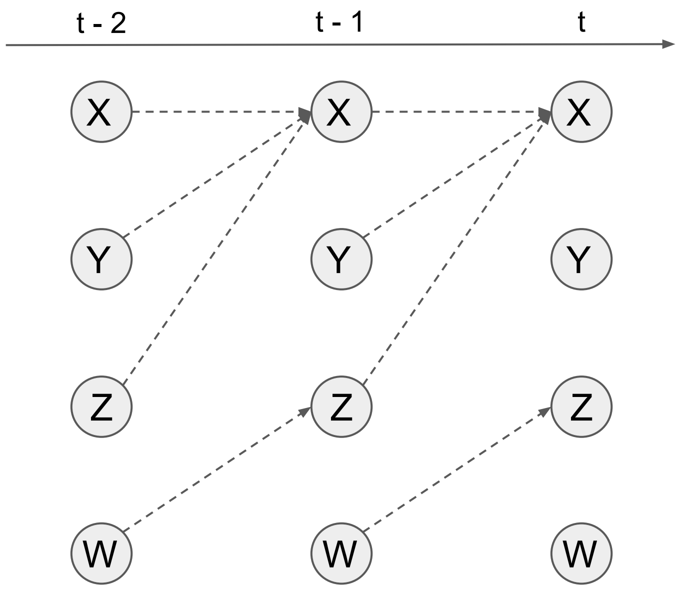
It consists of two main steps:

- **PC algorithm**
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- **MCI test**
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t | \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$

Key parameter: τ maximum time delay



Causal Discovery for Time-series Data

PCMCI algorithm

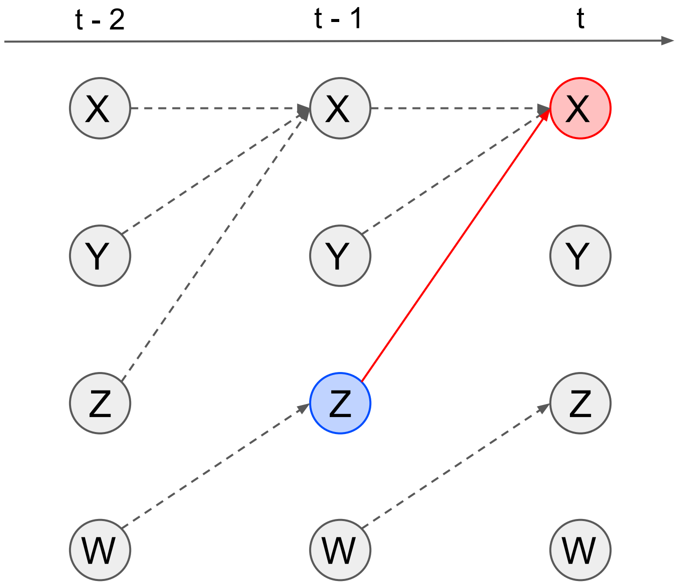
It consists of two main steps:

- **PC algorithm**
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- **MCI test**
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$\boxed{X^i_{t-\tau}} \perp\!\!\!\perp \boxed{X^j_t} | \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$

Key parameter: τ maximum time delay



Causal Discovery for Time-series Data
PCMCI algorithm

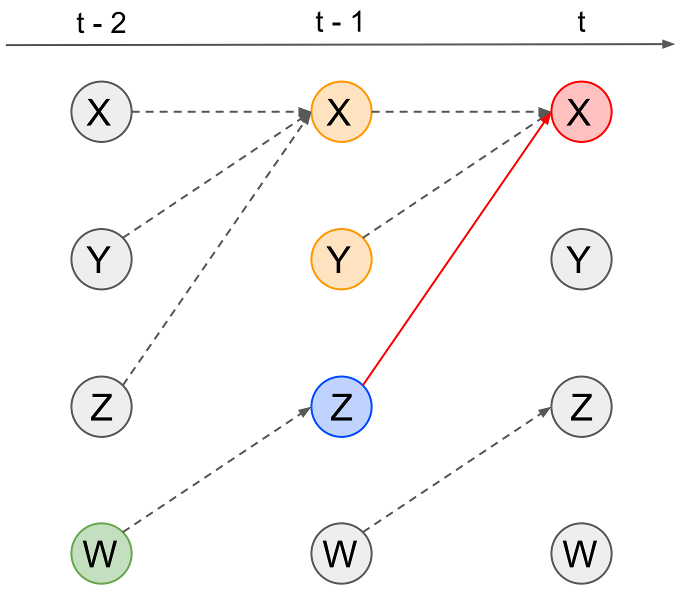
It consists of two main steps:

- **PC algorithm**
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- **MCI test**
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t \mid \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$

Key parameter: τ maximum time delay



Causal Discovery for Time-series Data

PCMCI algorithm

$$\begin{cases} X_t^0 = 0.4(X_{t-1}^1)^2 + \eta_t^0 \\ X_t^1 = \eta_t^1 \\ X_t^2 = 0.3(X_{t-2}^1)^2 + \eta_t^2 \end{cases}$$

```
random_state = np.random.default_rng(seed=42)
data = random_state.standard_normal((500, 3))
for t in range(1, 500):
    data[t, 0] += 0.4*data[t-1, 1]**2
    data[t, 2] += 0.3*data[t-2, 1]**2
var_names = [r'$X^0$', r'$X^1$', r'$X^2$']

dataframe = pp.DataFrame(data, var_names=var_names)
```

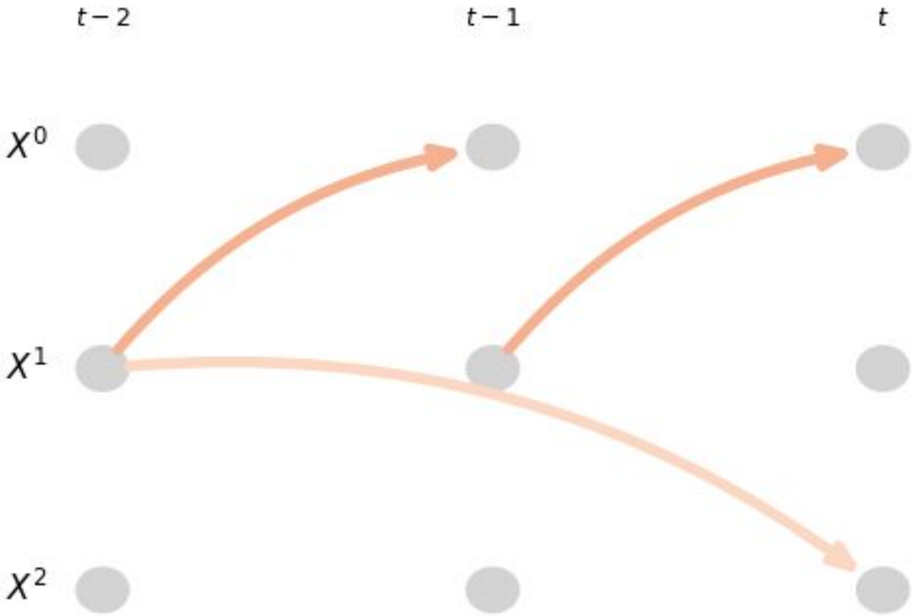
```
gpdc = GPDC(significance='analytic', gp_params=None)
pcmci_gpdc = PCMCI(
    dataframe=dataframe,
    cond_ind_test=gpdc,
    verbosity=0)
```

```
results = pcmci_gpdc.run_pcmci(tau_max=2, pc_alpha=0.1, alpha_level = 0.01)
tp.plot_graph(
    val_matrix=results['val_matrix'],
    graph=results['graph'],
    var_names=var_names,
    show_colorbar=False,
    )
```

Causal Discovery for Time-series Data

PCMCI algorithm

$$\begin{cases} X_t^0 = 0.4(X_{t-1}^1)^2 + \eta_t^0 \\ X_t^1 = \eta_t^1 \\ X_t^2 = 0.3(X_{t-2}^1)^2 + \eta_t^2 \end{cases}$$



Outline

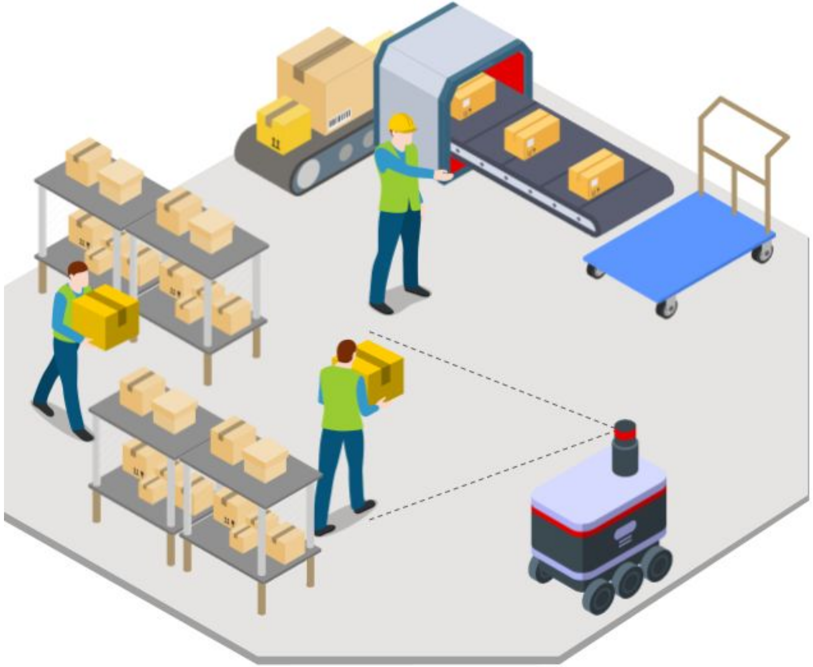
- Causal Discovery for Time-series Data
  - PCMCI algorithm

- **Robotics Applications**
  - F-PCMCI algorithm
  - ROS-Causal

Robotics Applications

Two main challenges in robotics:

- execution time of the causal discovery analysis
- conduct the causal discovery analysis online



Outline

- Causal Discovery for Time-series Data
  - PCMCI algorithm

- Robotics Applications
  - **F-PCMCI algorithm**
  - ROS-Causal

Robotics Applications
F-PCMCI algorithm

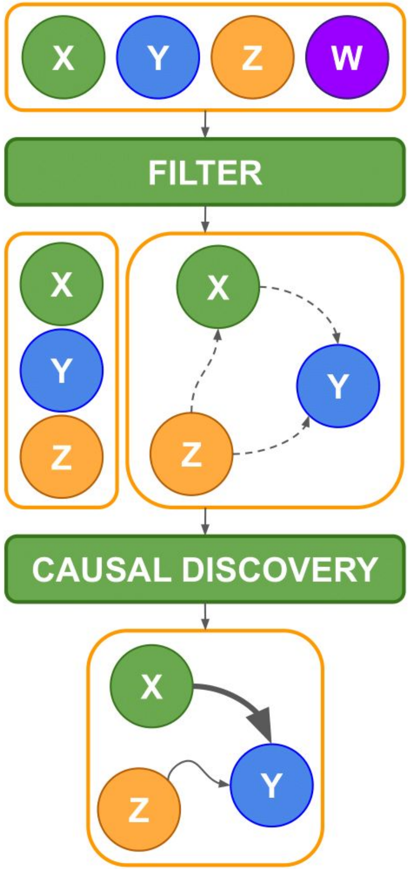
PCMCI computational cost depends on:

- the length of the time-series
- the number of variables

F-PCMCI steps:

- takes in input a prefixed set of variables
- the Transfer Entropy-based filter analyses and removes irrelevant variables (e.g., constants or isolated ones). The reduced variable set is used to create a hypothetical causal model
- the hypothetical causal model needs to be validated by a proper causal analysis, which is performed by PCMCI

This strategy enables **faster** and **more accurate** causal discovery



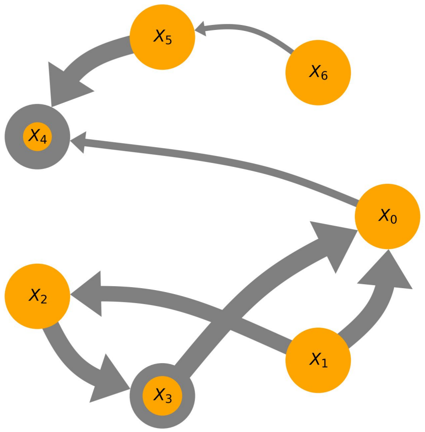
Castri, Luca, et al. "Enhancing Causal Discovery from Robot Sensor Data in Dynamic Scenarios." (2023).

Robotics Applications

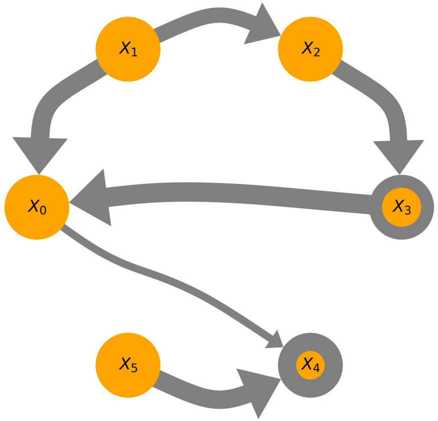
F-PCMCI algorithm

$$
\begin{cases}
x_0(t) = 2x_1(t-1) + 3x_3(t-1) + \eta_0 \\
x_1(t) = \eta_1 \\
x_2(t) = 1.1x_1(t-1)^2 + \eta_2 \\
x_3(t) = x_3(t-1) \cdot x_2(t-1) + \eta_3 \\
x_4(t) = x_4(t-1) + x_5(t-1) \cdot x_0(t-1) + \eta_4 \\
x_5(t) = \eta_5 \\
x_6(t) = \eta_6
\end{cases}
$$

PCMCI



F-PCMCI



Castri, Luca, et al. "Enhancing Causal Discovery from Robot Sensor Data in Dynamic Scenarios." (2023).

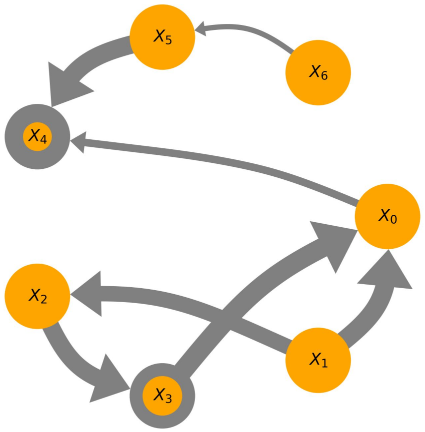
Robotics Applications

F-PCMCI algorithm

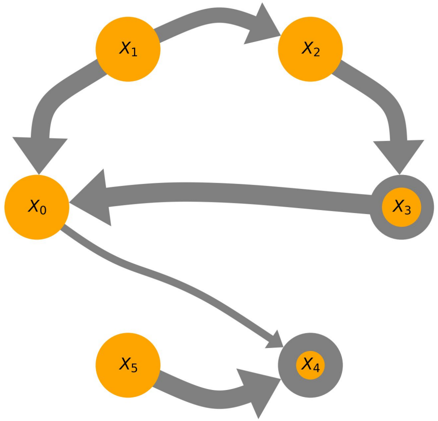
$$
\begin{cases}
x_0(t) = 2x_1(t-1) + 3x_3(t-1) + \eta_0 \\
x_1(t) = \eta_1 \\
x_2(t) = 1.1x_1(t-1)^2 + \eta_2 \\
x_3(t) = x_3(t-1) \cdot x_2(t-1) + \eta_3 \\
x_4(t) = x_4(t-1) + x_5(t-1) \cdot x_0(t-1) + \eta_4 \\
x_5(t) = \eta_5 \\
x_6(t) = \eta_6
\end{cases}
$$

Isolated

PCMCI



F-PCMCI



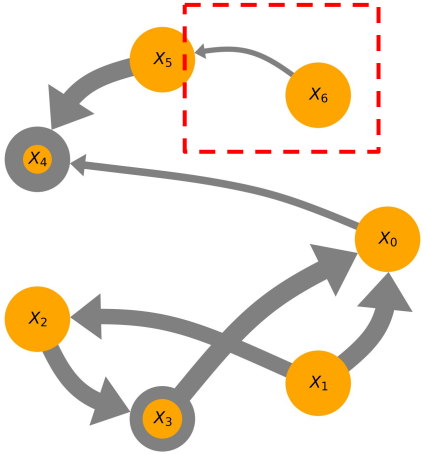
Castri, Luca, et al. "Enhancing Causal Discovery from Robot Sensor Data in Dynamic Scenarios." (2023).

Robotics Applications

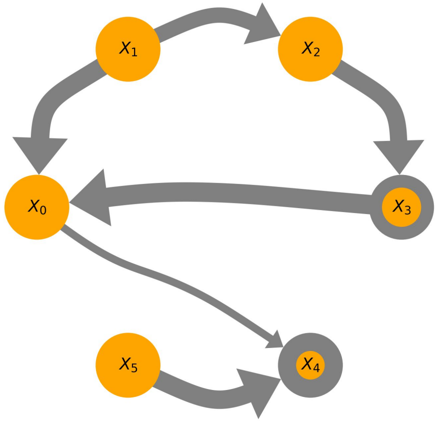
F-PCMCI algorithm

$$
\begin{cases}
x_0(t) = 2x_1(t-1) + 3x_3(t-1) + \eta_0 \\
x_1(t) = \eta_1 \\
x_2(t) = 1.1x_1(t-1)^2 + \eta_2 \\
x_3(t) = x_3(t-1) \cdot x_2(t-1) + \eta_3 \\
x_4(t) = x_4(t-1) + x_5(t-1) \cdot x_0(t-1) + \eta_4 \\
x_5(t) = \eta_5 \\
x_6(t) = \eta_6 \quad \longleftarrow \text{Isolated}
\end{cases}
$$

PCMCI
~9mins



F-PCMCI
3mins



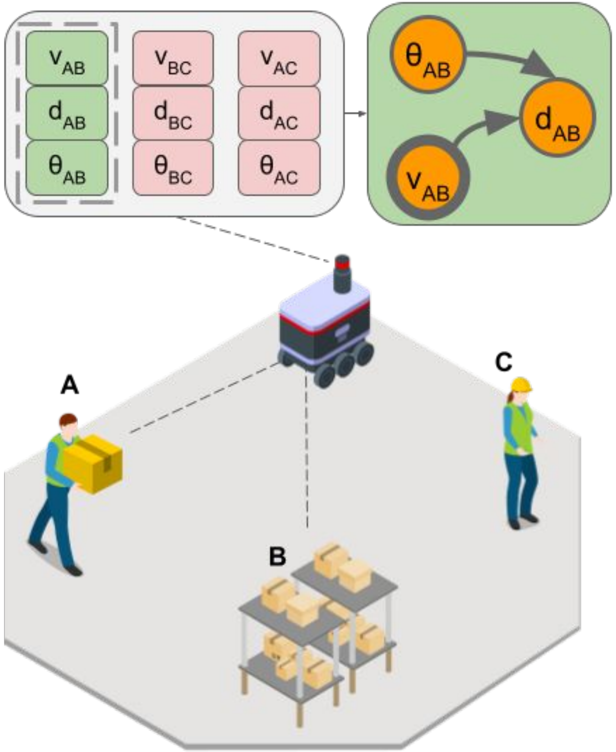
Castri, Luca, et al. "Enhancing Causal Discovery from Robot Sensor Data in Dynamic Scenarios." (2023).

Considering the interaction scenario modelled by three variables

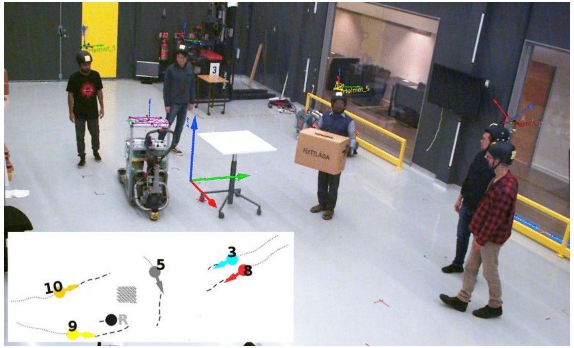
- v_{ij}: relative velocity between agent i and j
- d_{ij}: distance between agent i and j
- $theta_{ij}$: angle between agent i and j

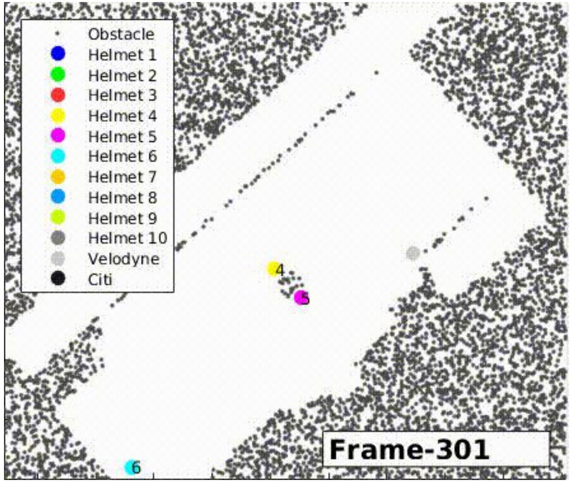
Are all the observable variables useful to understand the observed scenario?

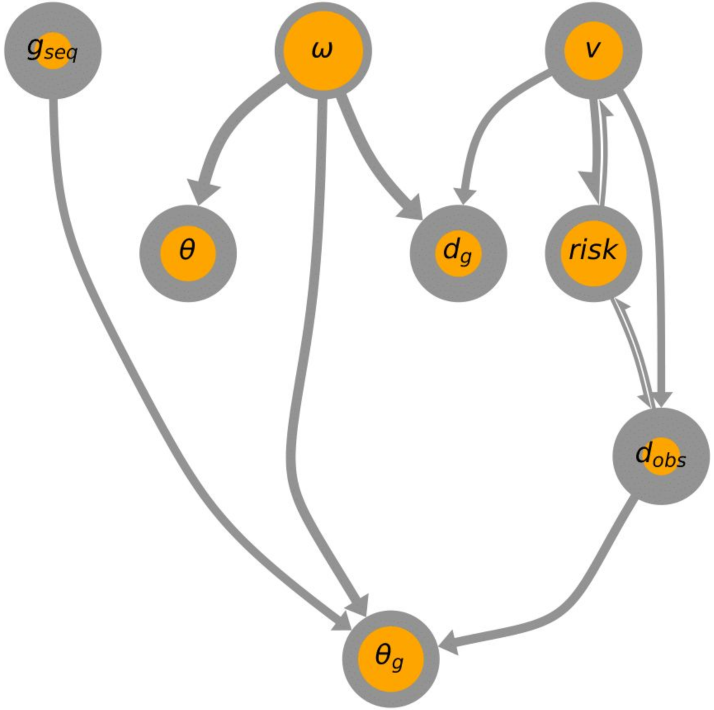
We need to filter the variables before conducting the causal analysis



Robotics Applications
F-PCMCI algorithm

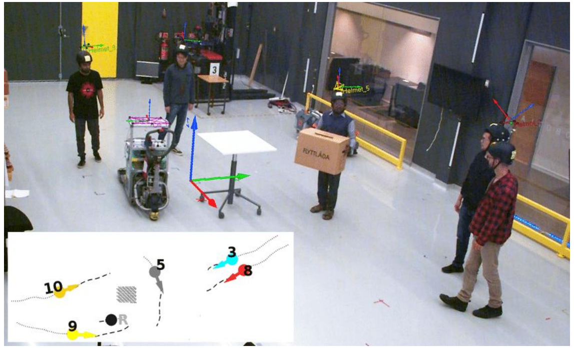


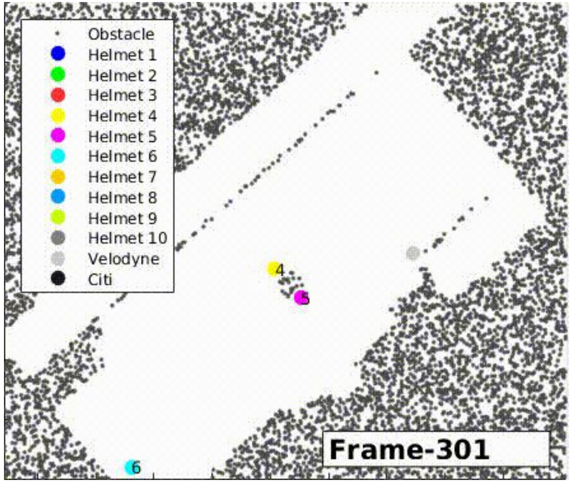


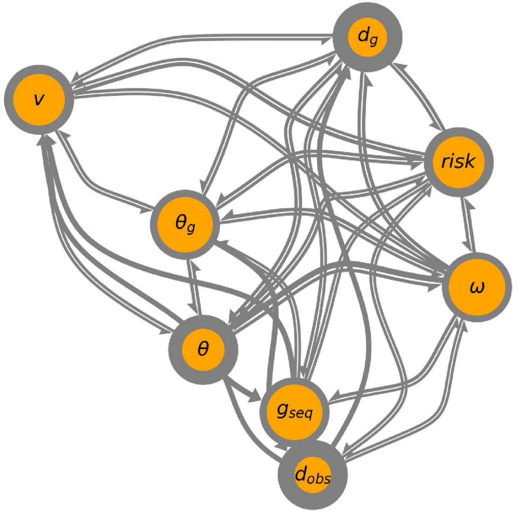


Castri, Luca, et al. "Enhancing Causal Discovery from Robot Sensor Data in Dynamic Scenarios." (2023).

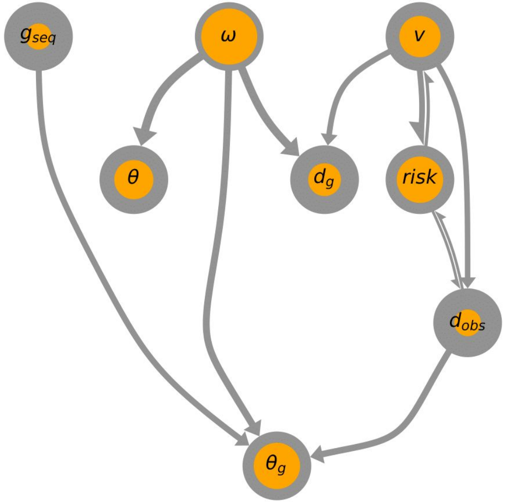
Robotics Applications
F-PCMCI algorithm







PCMCI ~80mins



F-PCMCI ~18mins

Castri, Luca, et al. "Enhancing Causal Discovery from Robot Sensor Data in Dynamic Scenarios." (2023).

Outline

- Causal Discovery for Time-series Data
  - PCMCI algorithm

- Robotics Applications
  - F-PCMCI algorithm
  - **ROS-Causal**

What is Robot Operating System (ROS)?

people tracker

navigation stack

...

What is Robot Operating System (ROS)?

ROS

people tracker

navigation stack

...

What is Robot Operating System (ROS)?

ROS

people tracker

navigation stack

...

What is Robot Operating System (ROS)?

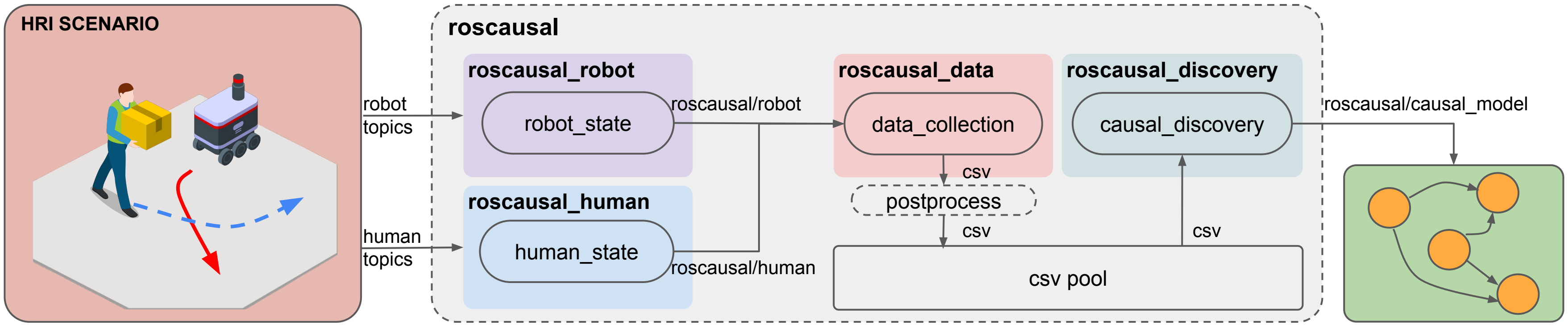
ROS

people tracker

navigation stack

ROS-Causal

Robotics Applications
ROS-Causal

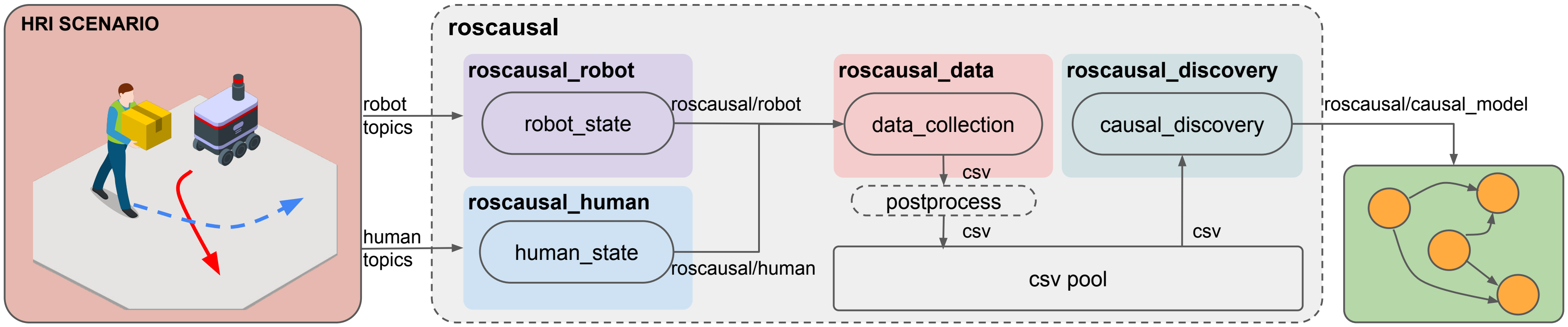


- ROS-Causal extracts and collects data from a HRI scenario, such as agents’ trajectories, and performs causal analysis on the collected data in a batched manner. It is composed by four different rosnodes:
  - roscausal_robot
  - roscausal_human
  - roscausal_data
  - roscausal_discovery

Castri, Luca, et al. "ROS-Causal: A ROS-based Causal Analysis Framework for Human-Robot Interaction Applications" (2024).

Robotics Applications

ROS-Causal



roscausal_robot: collects data from several rostopics related to the robot (e.g., position, velocity, target position, etc.), and merge them into a single rostopic: *roscausal/robot*

roscausal_human: collects data from several rostopics related to the human (e.g., position, velocity, target position, etc.), and merge them into a single rostopic: *roscausal/human*

roscausal_data: subscribes to the topics */roscausal/robot* and */roscausal/human* and begins collecting data in a CSV file. Once the desired time-series length (rosparam) is reached, the node provides the option to post-process the data and finally saves the CSV file into a designated folder.

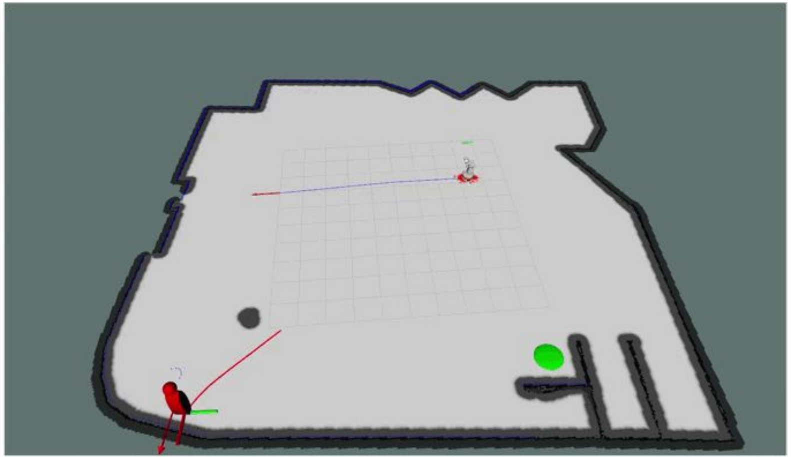
roscausal_discovery: performs causal discovery analysis on the collected data and publishes the result on the *roscausal/causal_model* rostopic. So far, it incorporates two causal discovery methods: PCMCI and F-PCMCI.

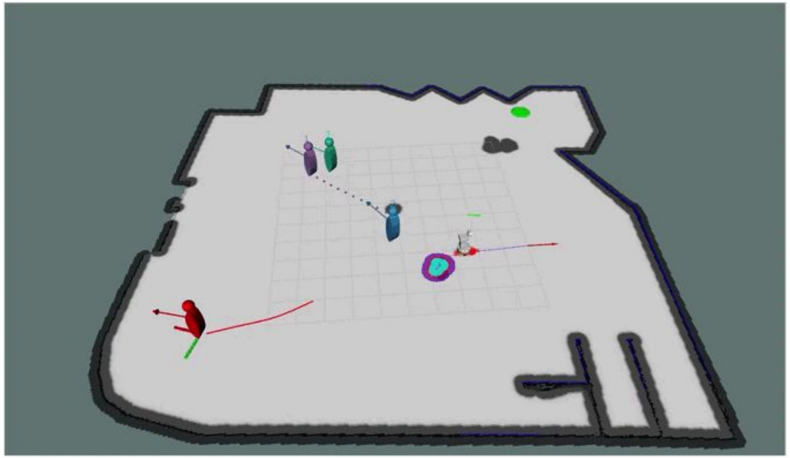
Castri, Luca, et al. "ROS-Causal: A ROS-based Causal Analysis Framework for Human-Robot Interaction Applications" (2024).

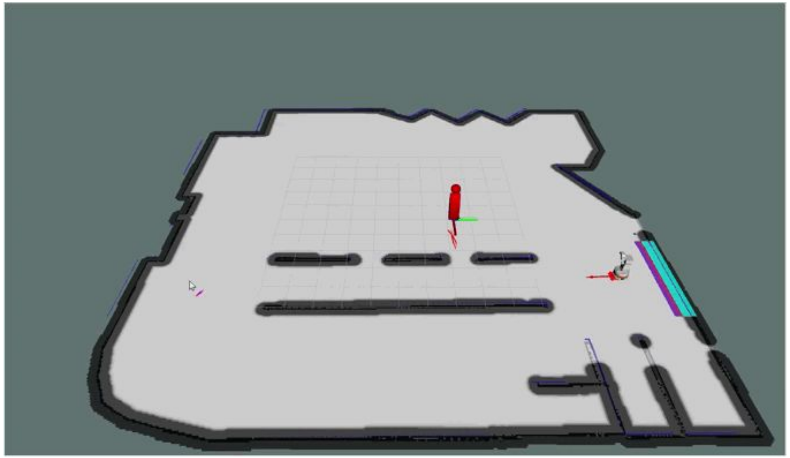
ROS-Causal_HRISim

HRI simulator involving:

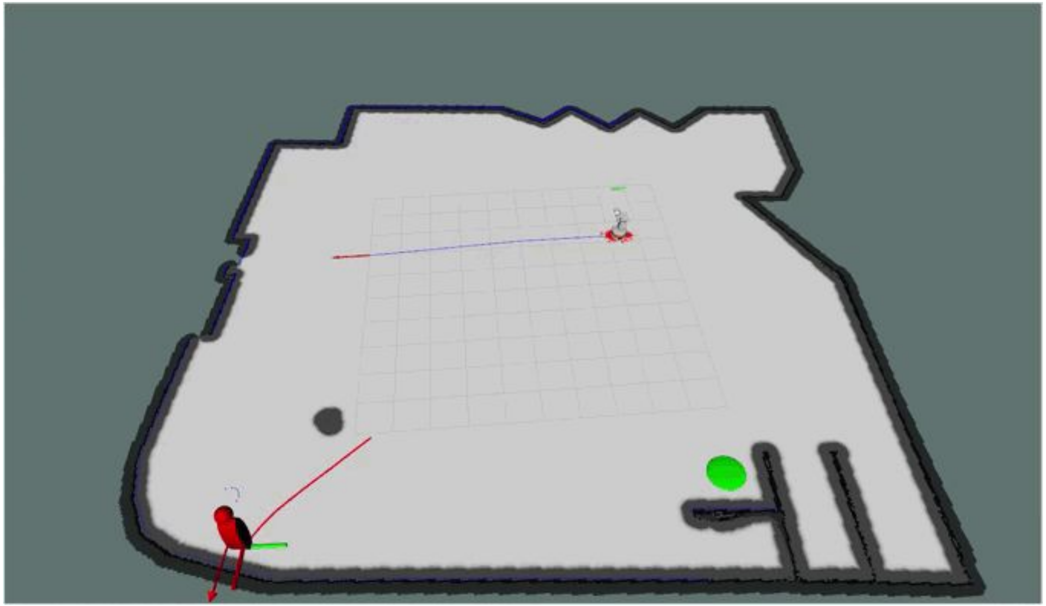
- TIAGo robot
- pedestrians

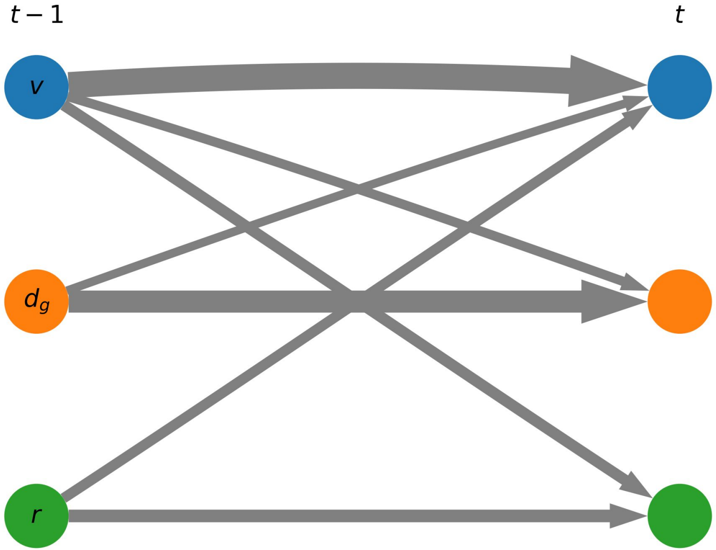






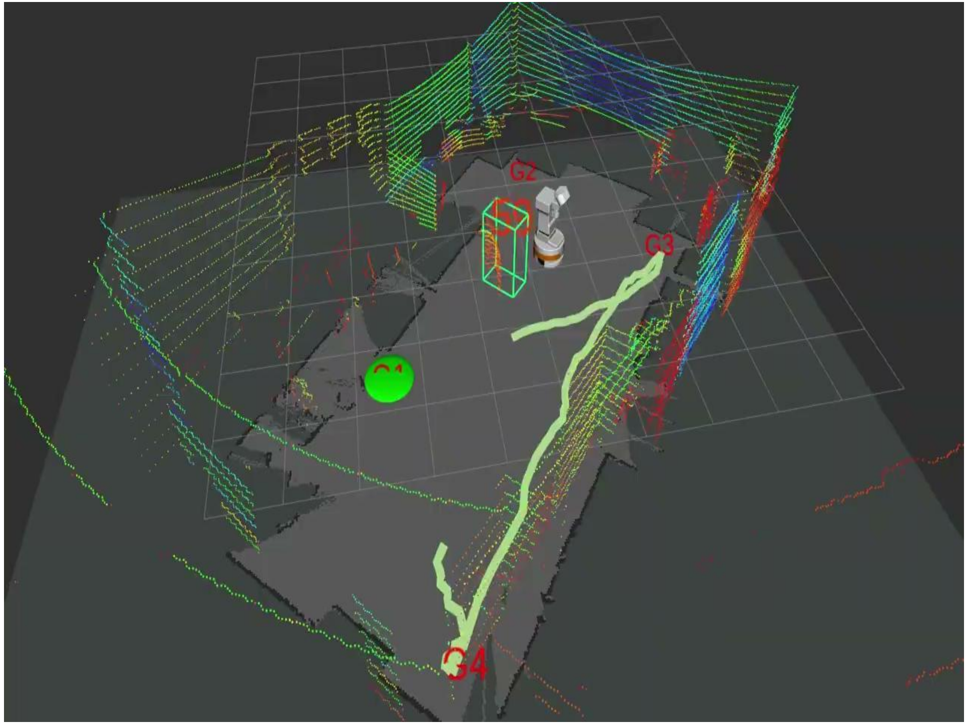
Robotics Applications
ROS-Causal

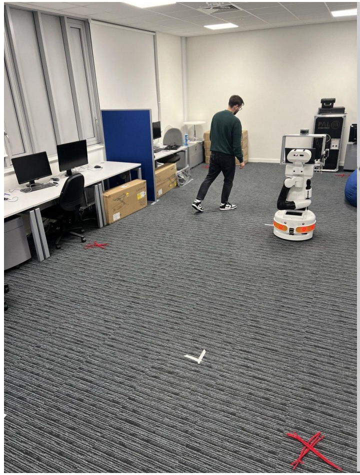


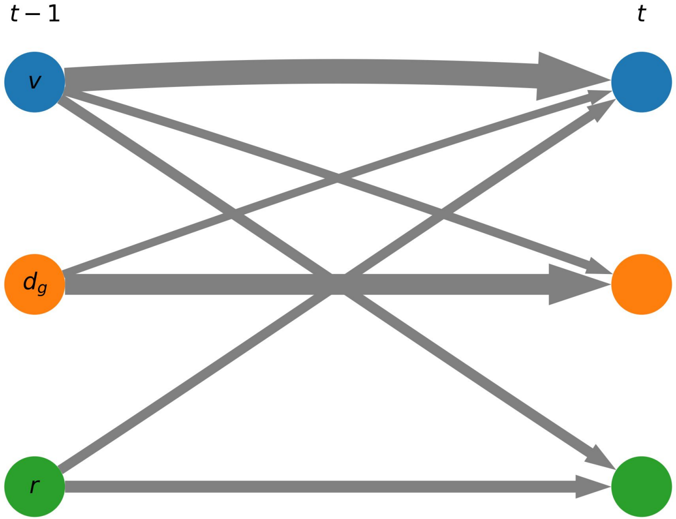


Robotics Applications

ROS-Causal

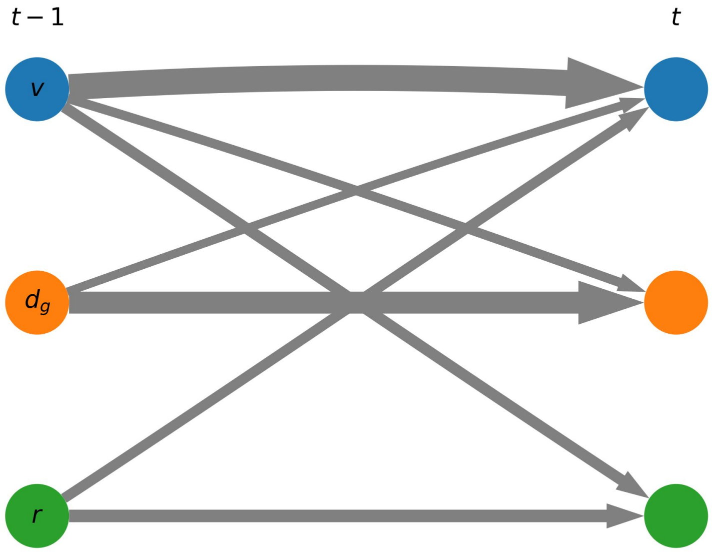






Castri, Luca, et al. "Experimental Evaluation of ROS-Causal in Real-World Human-Robot Spatial Interaction Scenarios" (2024).

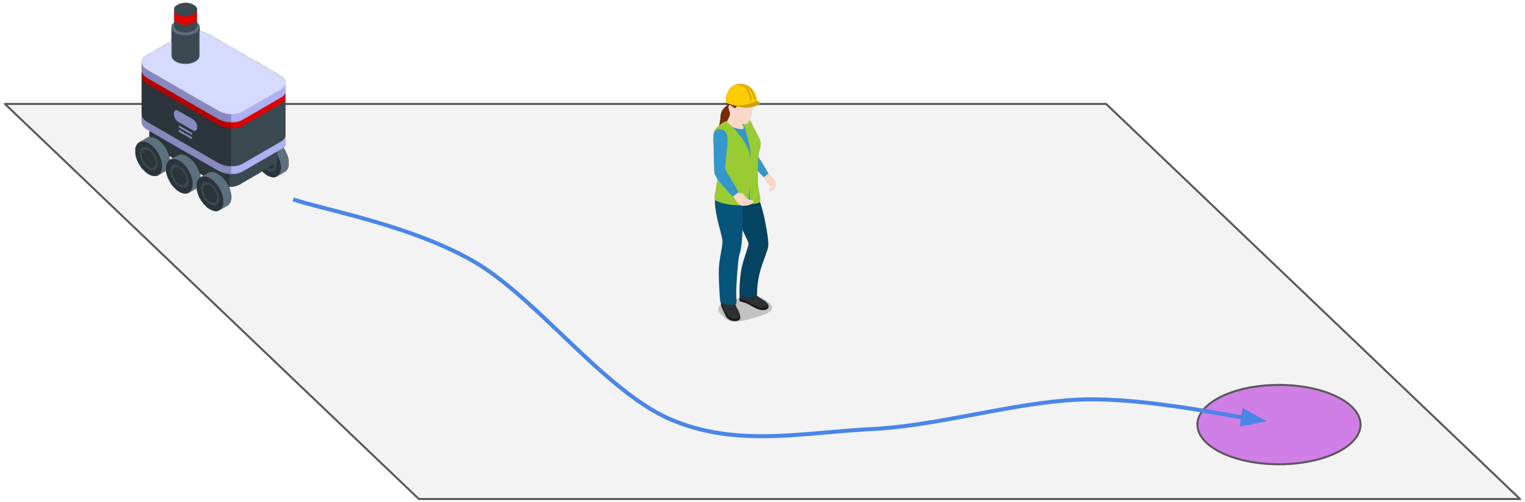
- What can a reconstructed causal model be useful for?
  - Prediction
  - Motion planning
  - Decision making



Robotics Applications
ROS-Causal

- What can a reconstructed causal model be useful for?
    - Prediction
    - Motion planning
    - Decision making

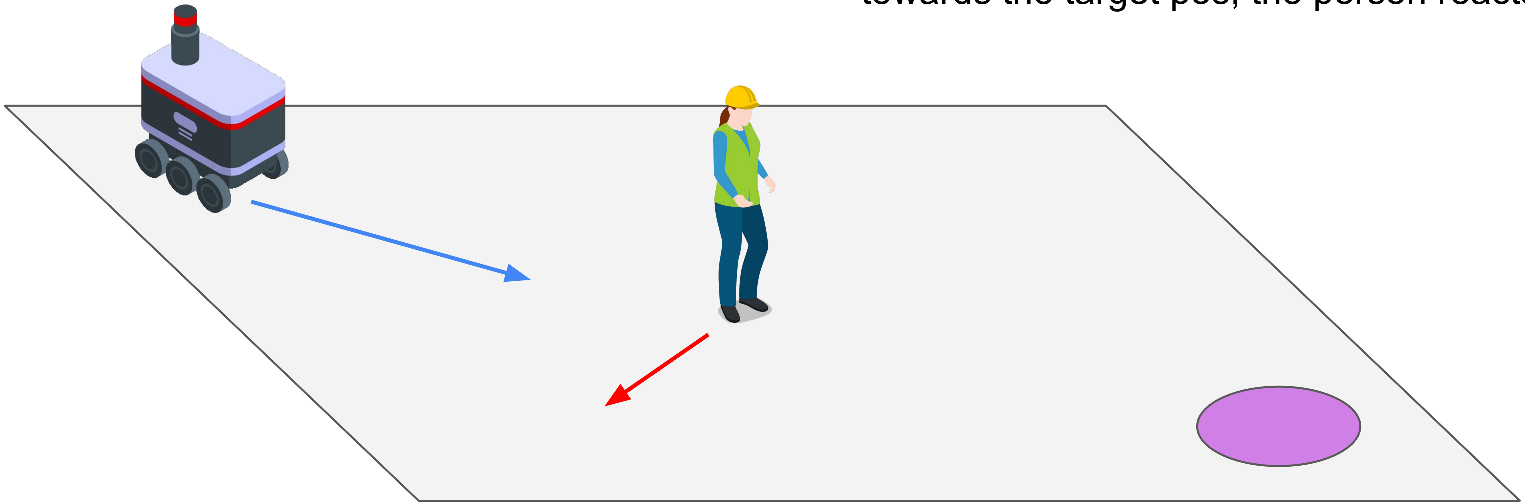
Trajectory generated by a planner not
accounting for human reaction to robot's actions



- What can a reconstructed causal model be useful for?
  - Prediction
  - Motion planning
  - Decision making

Trajectory generated by a planner not accounting for human reaction to robot's actions

But
What if, as soon as the robot starts moving towards the target pos, the person reacts?

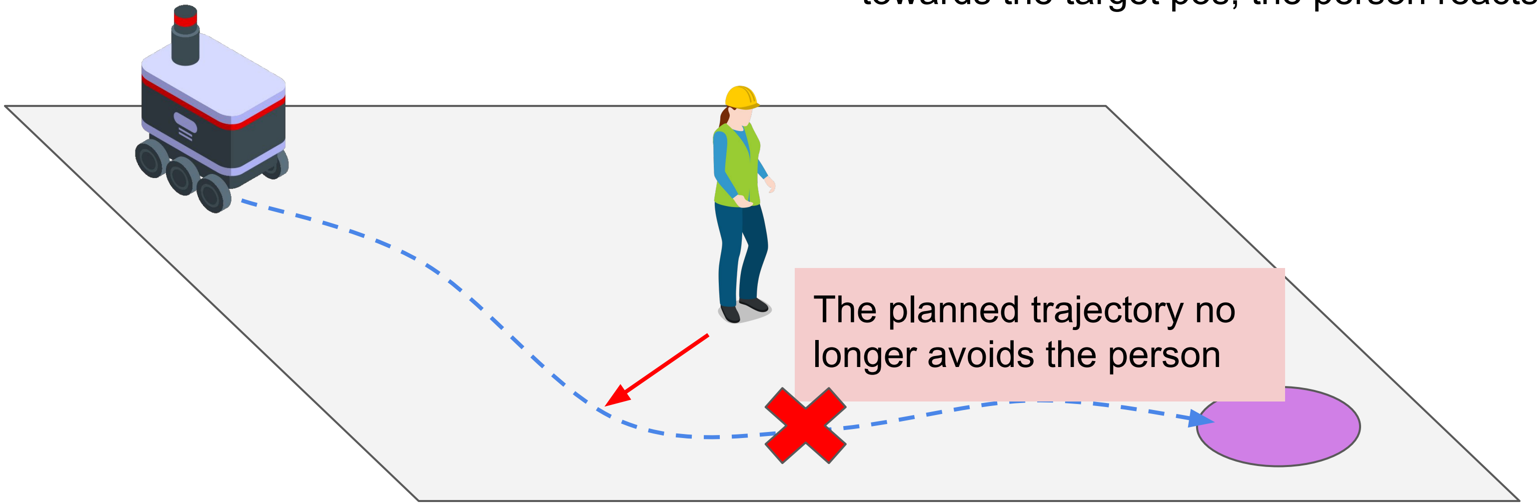


Robotics Applications
ROS-Causal

- What can a reconstructed causal model be useful for?
  - Prediction
  - Motion planning
  - Decision making

Trajectory generated by a planner not accounting for human reaction to robot's actions

But
What if, as soon as the robot starts moving towards the target pos, the person reacts?

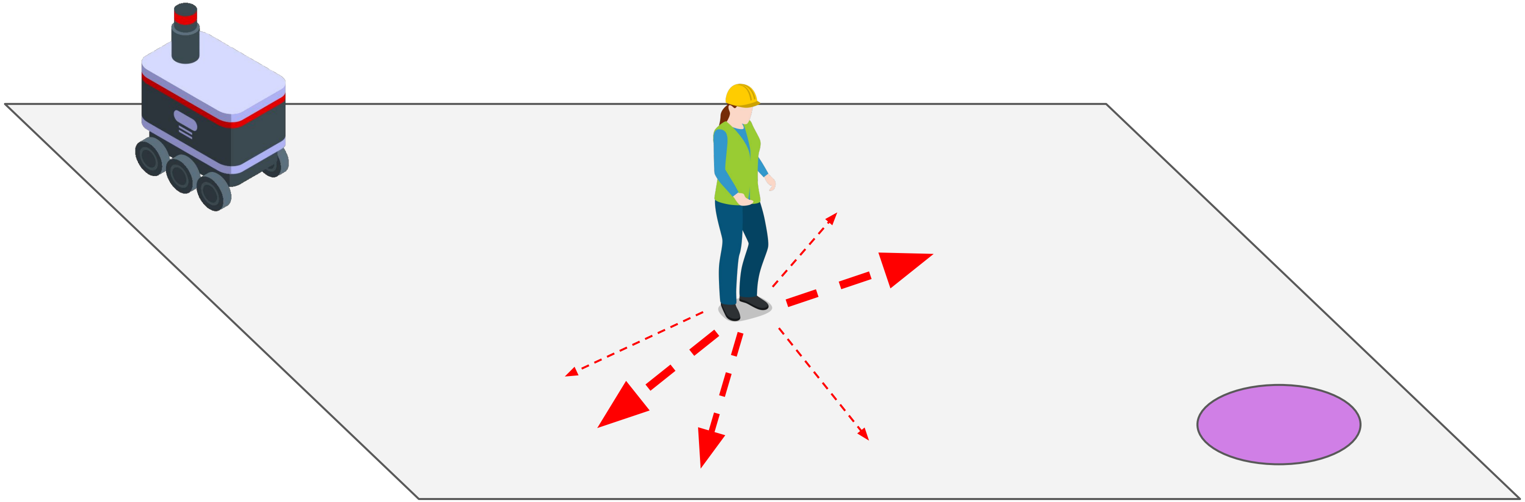


Robotics Applications
ROS-Causal

- What can a reconstructed causal model be useful for?
    - Prediction
    - Motion planning
    - Decision making

Trajectory generated by a planner not accounting for human reaction to robot's actions

Knowing the causal model
- can facilitate the prediction of the person spatial behaviours

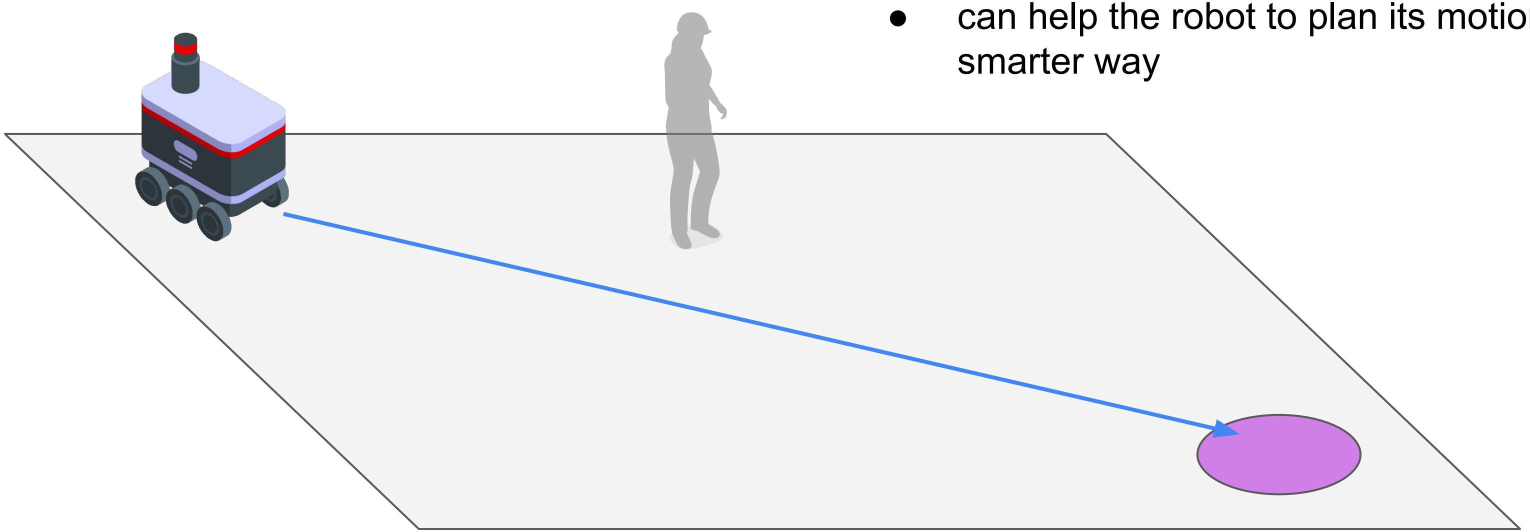


Robotics Applications

ROS-Causal

- What can a reconstructed causal model be useful for?
    - Prediction
    - Motion planning
    - Decision making

Trajectory generated by a planner not accounting for human reaction to robot's actions



Knowing the causal model

- can facilitate the prediction of the person spatial behaviours
- can help the robot to plan its motion in a smarter way

Reference

- Runge, J., Nowack, P., Kretschmer, M., Flaxman, S. and Sejdinovic, D., 2019. Detecting and quantifying causal associations in large nonlinear time series datasets. Science advances, 5(11).
- Castri, L., Mghames, S., Hanheide, M. and Bellotto, N., 2023. Enhancing Causal Discovery from Robot Sensor Data in Dynamic Scenarios. In 2nd Conference on Causal Learning and Reasoning.
- Castri, L., Beraldo, G., Mghames, S., Hanheide, M. and Bellotto, N., 2024. ROS-Causal: A ROS-based Causal Analysis Framework for Human-Robot Interaction Applications, Workshop on Causal Learning for Human-Robot Interaction (Causal-HRI), ACM/IEEE International Conference on Human-Robot Interaction (HRI).
- Castri, L., Beraldo, G., Mghames, S., Hanheide M. and Bellotto, N. 2024. Experimental Evaluation of ROS-Causal in Real-World Human-Robot Spatial Interaction Scenarios. In IEEE International Conference on Robot and Human Interactive Communication (RO-MAN).

PCMCI **F-PCMCI** **ROS-Causal**

Thank you, questions?

Research topics

- Conditional independence test for mixed data (linear and nonlinear)

$$
\begin{cases}
X_t^0 = 0.2(X_{t-1}^1)^2 + \eta_t^0 \\
X_t^1 = \eta_t^1 \\
X_t^2 = \begin{cases} 0 & \text{if } X_{t-1}^1 \leq 0 \\ 1 & \text{if } 0 < X_{t-1}^1 \leq 10 \\ 2 & \text{if } X_{t-1}^1 > 10 \end{cases}
\end{cases}
$$

How can we test for independence between continuous and categorical data in the time-series domain?

- From tigramite (https://github.com/jakobrunge/tigramite): RegressionCI - mixed datasets with univariate discrete/categorical and (linear) continuous variables
- Zan, Lei, et al. "A conditional mutual information estimator for mixed data and an associated conditional independence test." Entropy 24.9 (2022): 1234.