





Causal Discovery for Time-Series Data

Outline

- **Causal Discovery for Time-series Data**
 - PCMCI algorithm
- Robotics Applications
 -  F-PCMCI algorithm
 -  CAnDOIT algorithm
 -  CausalFlow
 -  ROS-Causal

Causal Discovery for Time-series Data

The PC and FCI causal discovery methods work well with discrete/categorical data.

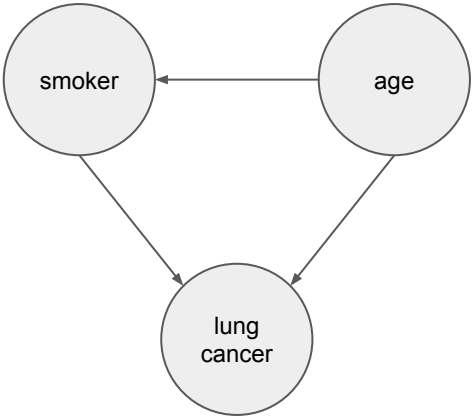
example

non-smoker **0** - smoker **1**

age under 50 **0** - age over 50 **1**

no lung cancer **0** - lung cancer **1**

Smoker	Age	Lung cancer
0	0	0
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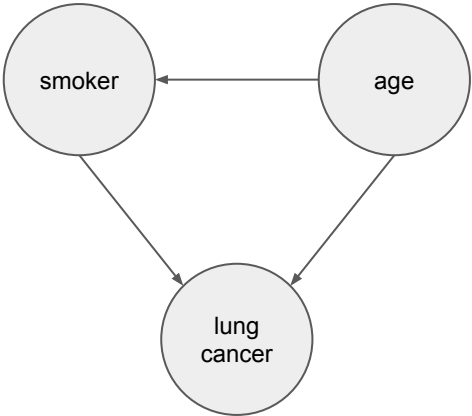
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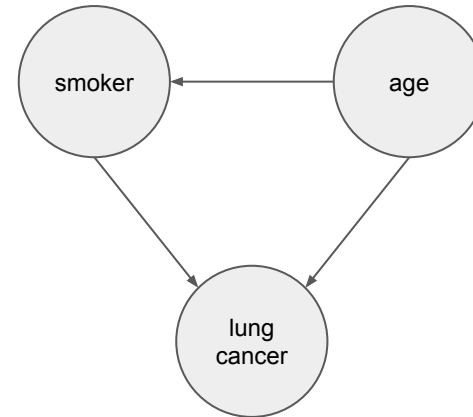
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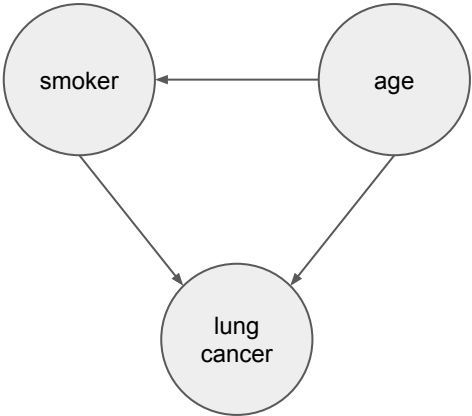
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The order is not important

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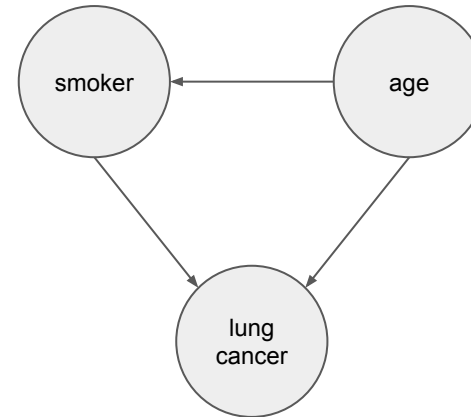
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What if our data is time-dependent?



Causal Discovery for Time-series Data

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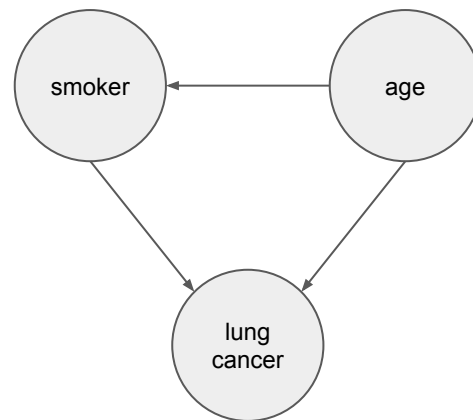
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We cannot only consider the contemporaneous relationships

Causal Discovery for Time-series Data

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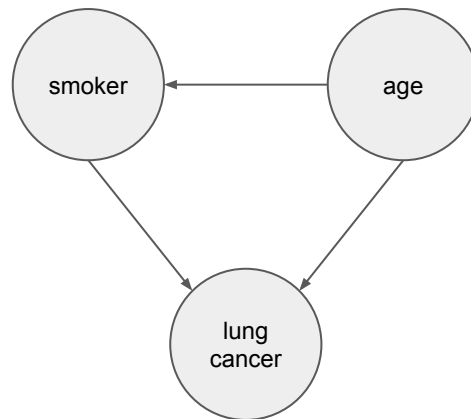
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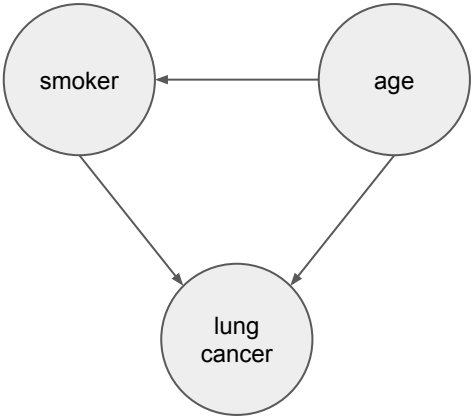
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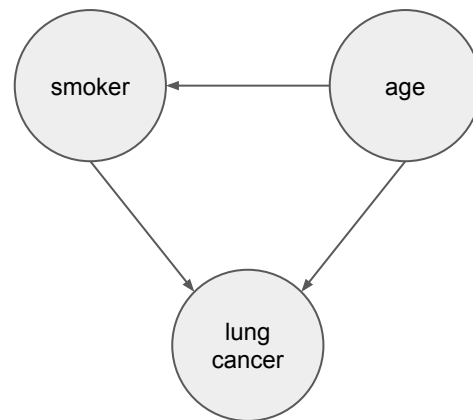
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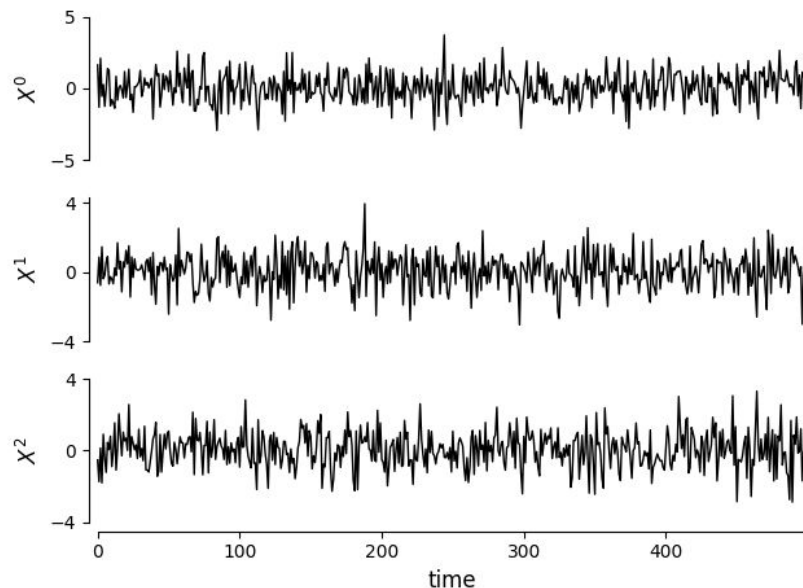
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What if we deal with time-series data?





PC/FCI is inappropriate to use with time series data due to:

- time ordering
- lagged dependencies
- high false positive rates due to the autocorrelation

$$\begin{cases} X_t^0 = 0.2(X_{t-1}^1)^2 + \eta_t^0 \\ X_t^1 = \eta_t^1 \\ X_t^2 = 0.3(X_{t-2}^1)^2 + \eta_t^2 \end{cases}$$



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Causal Discovery for Time-series Data

PCMCI algorithm

It consists of two main steps:

- **PC algorithm**
retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables
- **MCI test**
validates the structure found at the previous step by performing a false positive rate optimisation control

$$X_{t-\tau}^i \perp\!\!\!\perp X_t^j | \tilde{P}(X_{t-\tau}^i), \tilde{P}(X_t^j)$$

Key parameter: \mathcal{T} maximum time delay

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Z

W

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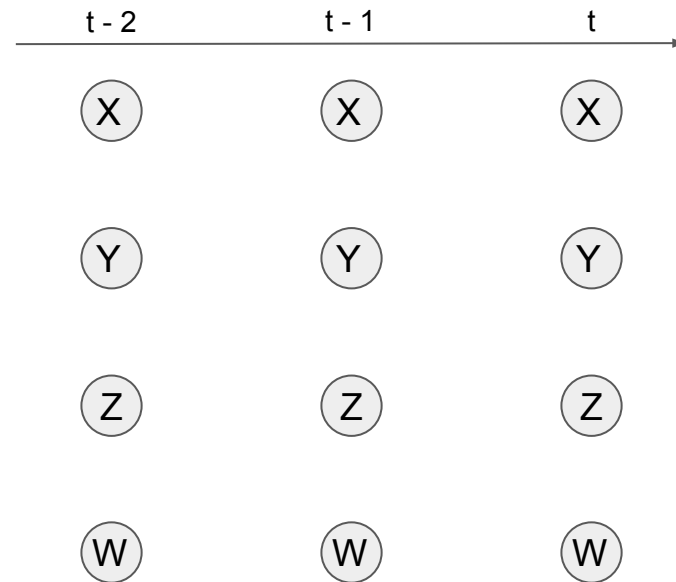
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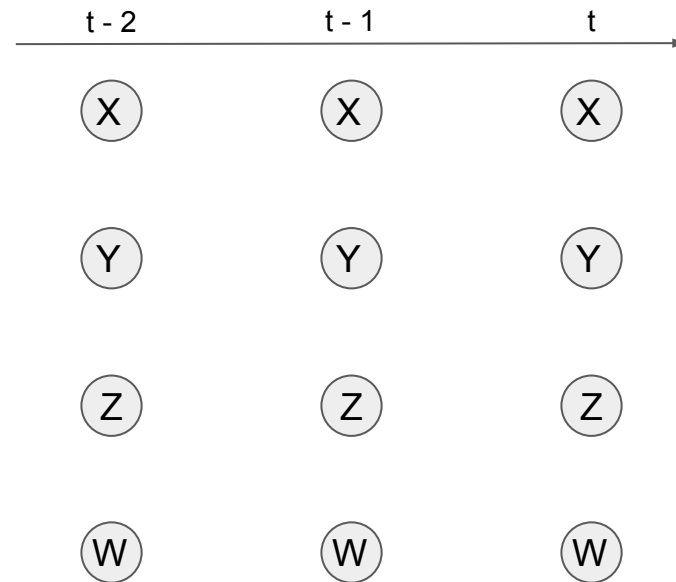
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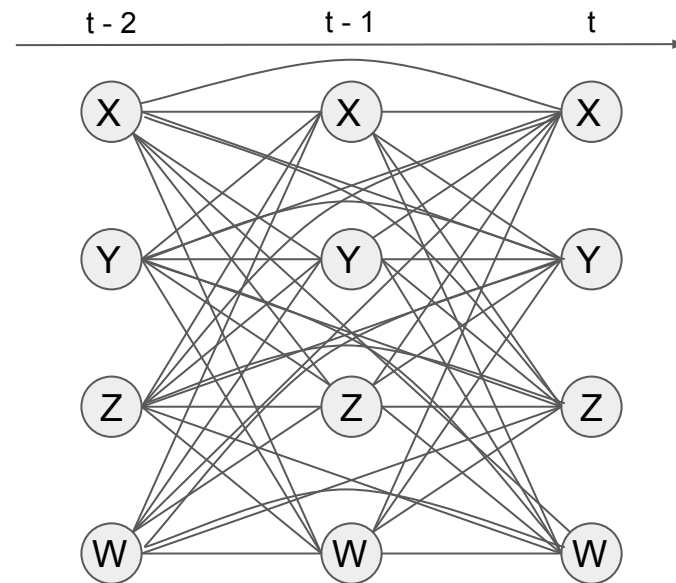
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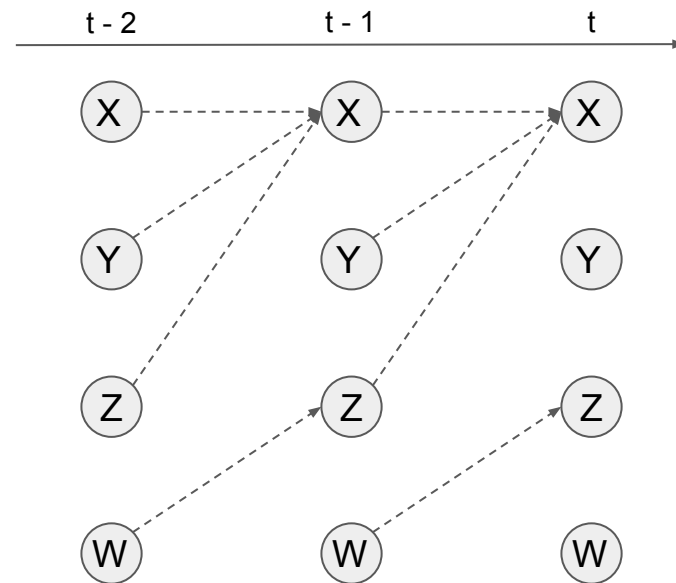
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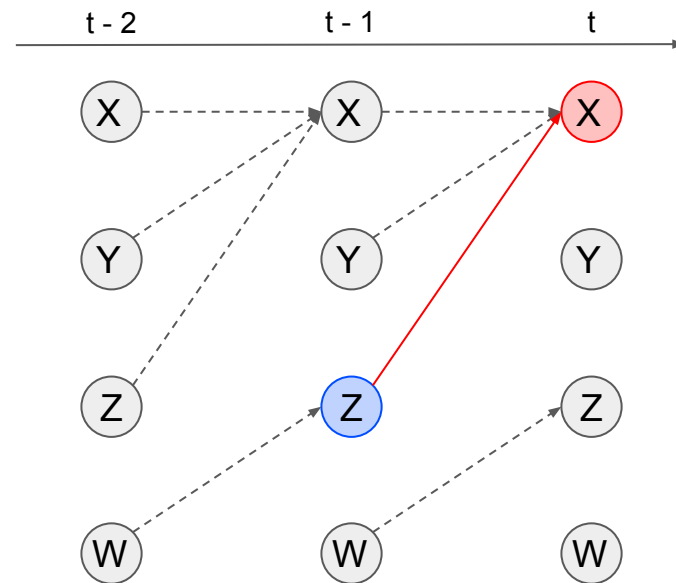
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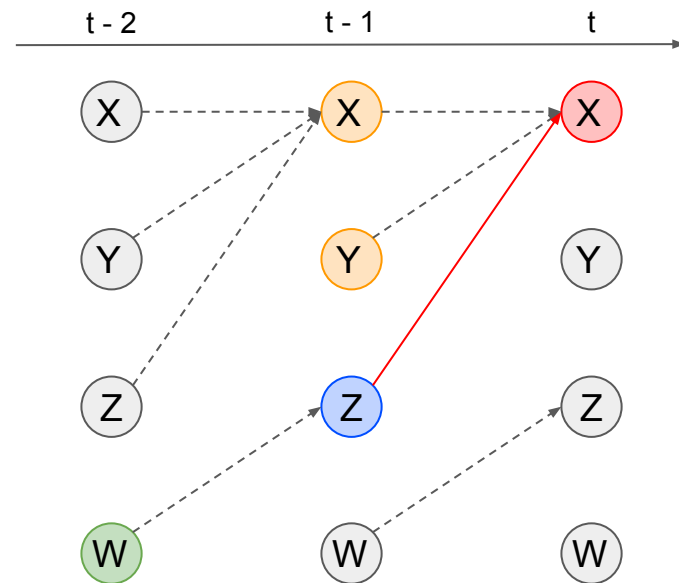
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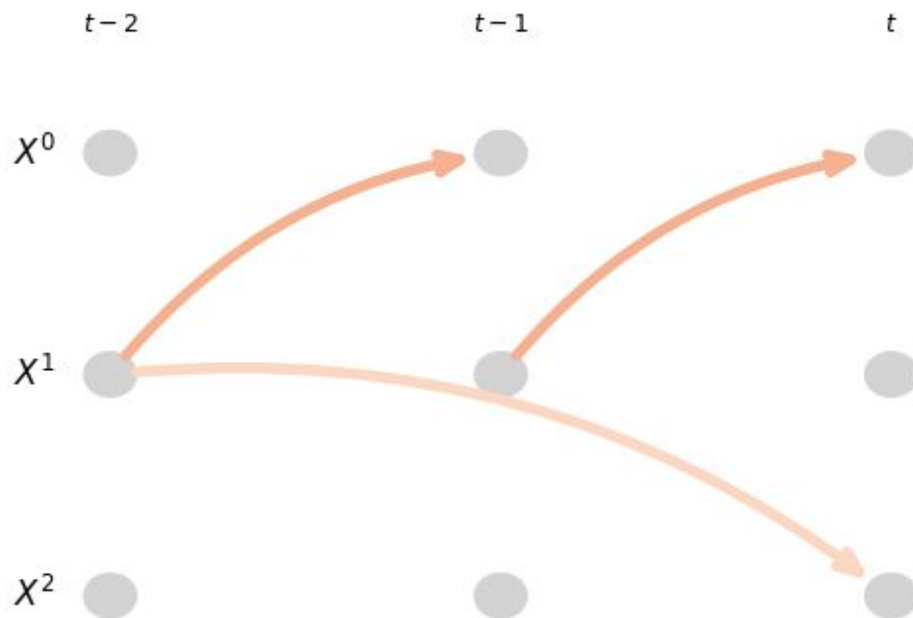
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Causal Discovery for Time-series Data

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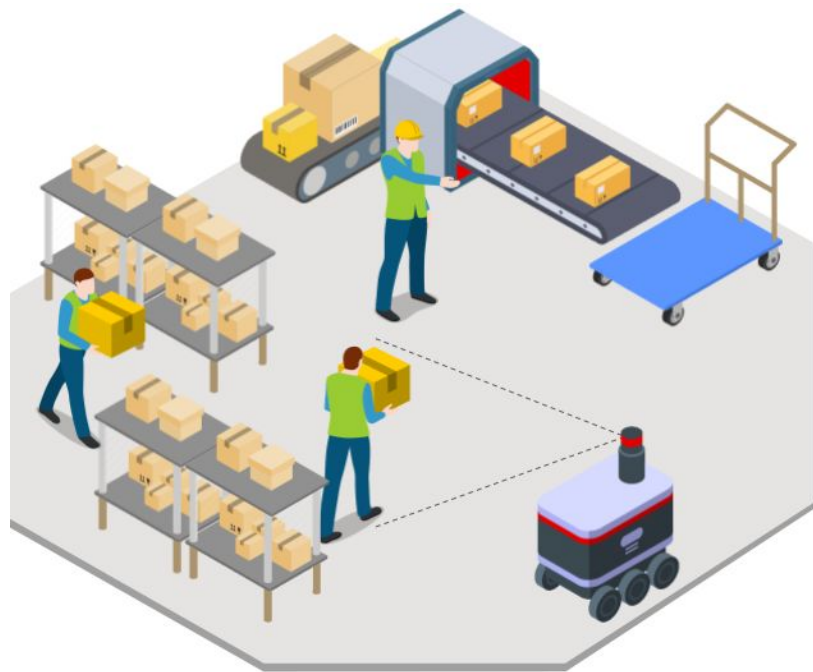
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Robotics Applications

Main challenges in robotics:

- execution time of the causal discovery analysis
- conduct causal discovery using data from observations and interventions
- conduct the causal discovery analysis online



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Robotics Applications

F-PCMCI algorithm

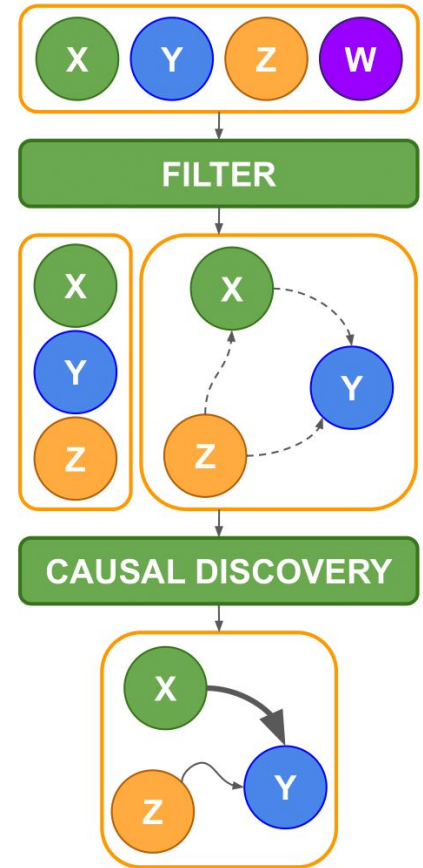
PCMCI computational cost depends on:

- the length of the time-series
- the number of variables

Filtered-PCMCI (**F-PCMCI**) steps:

- takes in input a prefixed set of variables
- the Transfer Entropy-based filter analyses and removes irrelevant variables (e.g., constants or isolated ones). The reduced variable set is used to create a hypothetical causal model
- the hypothetical causal model needs to be validated by a proper causal analysis, which is performed by PCMCI

This strategy enables **faster** and **more accurate** causal discovery



Robotics Applications

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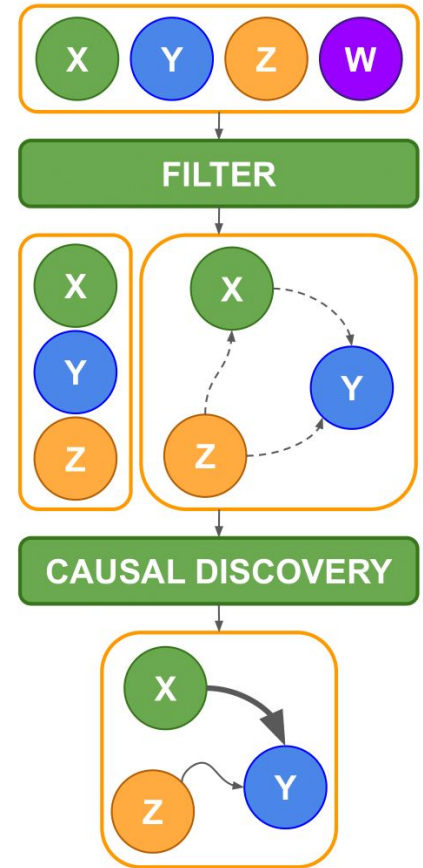
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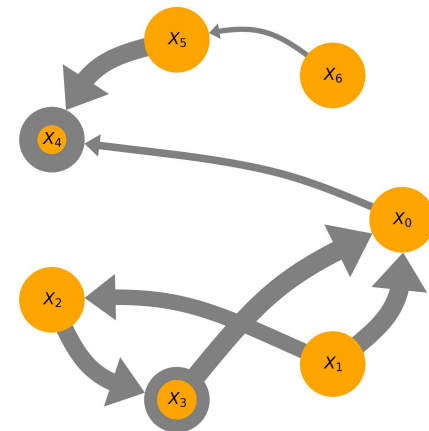


Robotics Applications

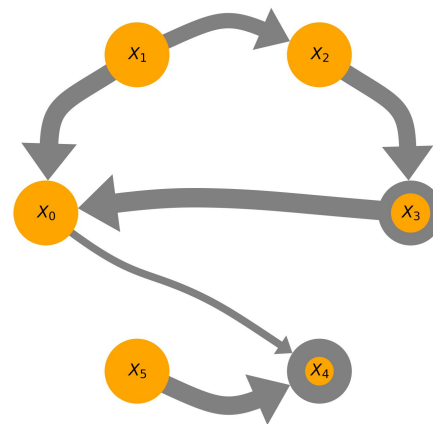
F-PCMCI algorithm

$$\left\{ \begin{array}{l} x_0(t) = 2x_1(t-1) + 3x_3(t-1) + \eta_0 \\ x_1(t) = \eta_1 \\ x_2(t) = 1.1x_1(t-1)^2 + \eta_2 \\ x_3(t) = x_3(t-1) \cdot x_2(t-1) + \eta_3 \\ x_4(t) = x_4(t-1) + x_5(t-1) \cdot x_0(t-1) + \eta_4 \\ x_5(t) = \eta_5 \\ x_6(t) = \eta_6 \end{array} \right.$$

PCMCI



F-PCMCI

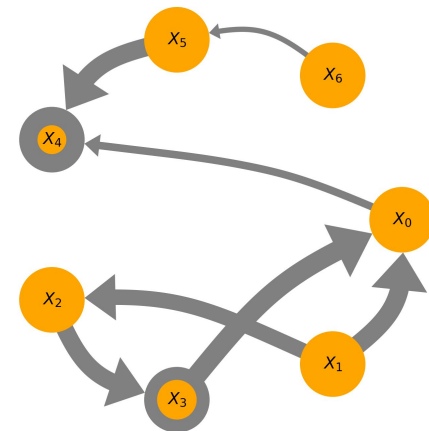


Robotics Applications

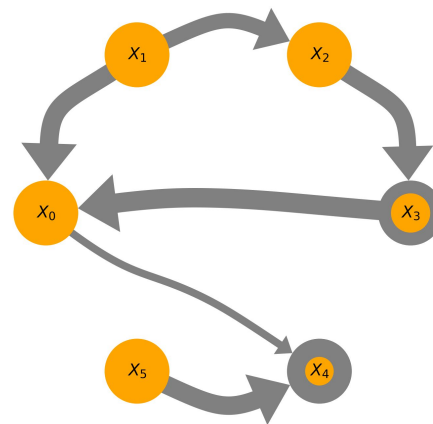
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PCMCI




F-PCMCI

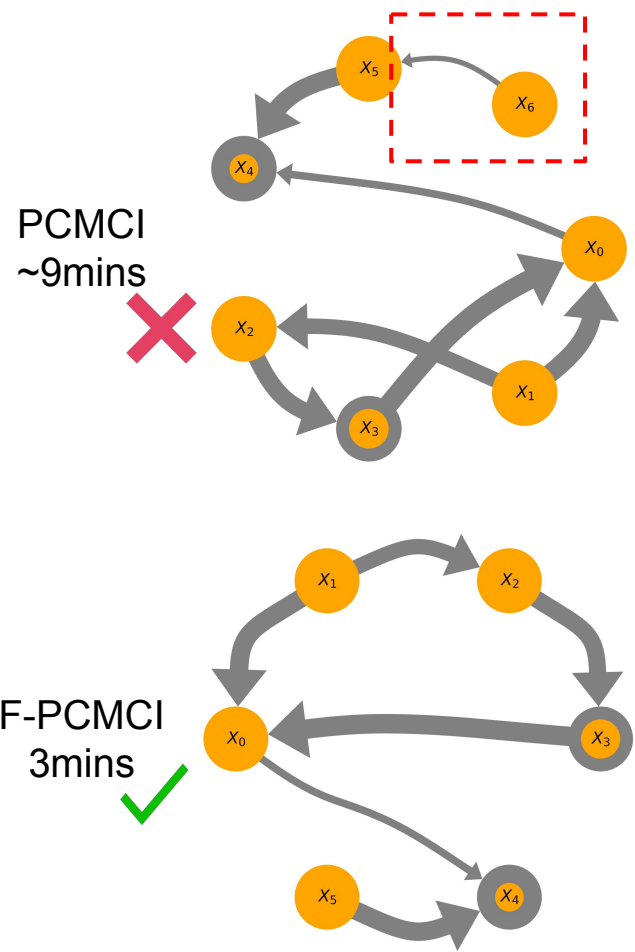


Robotics Applications

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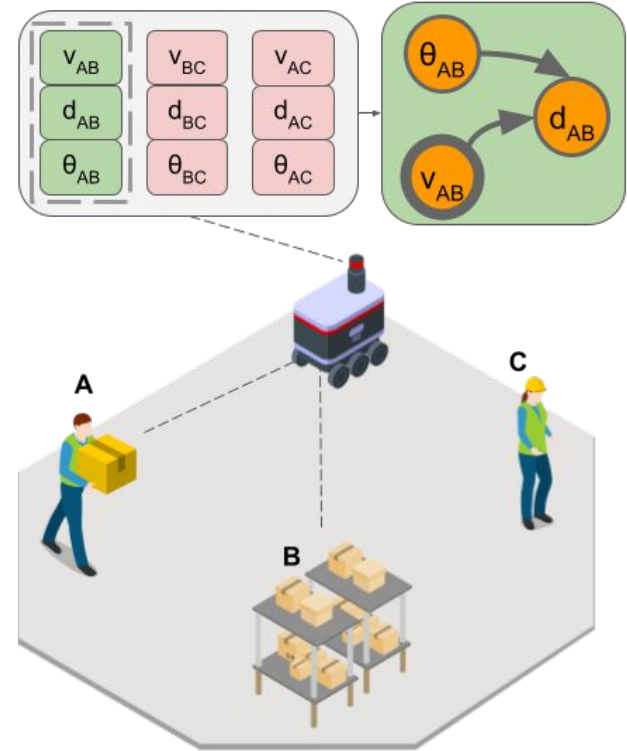
Robotics Applications

F-PCMRI algorithm

Considering the interaction scenario modelled by three variables

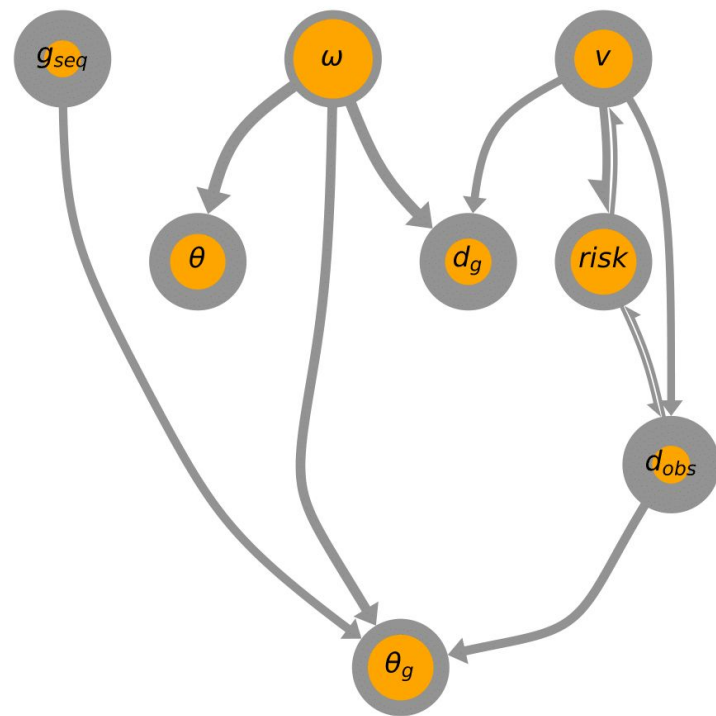
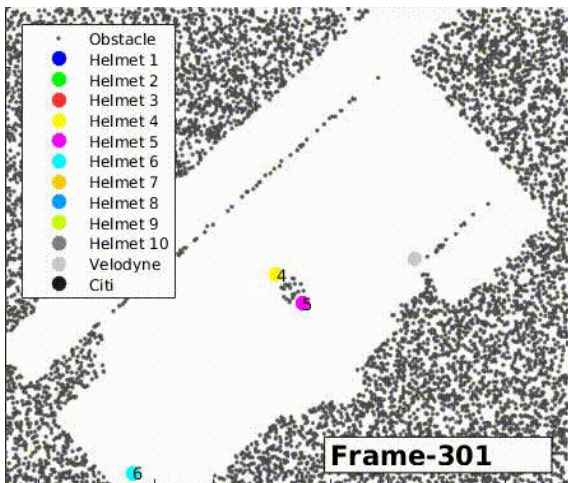
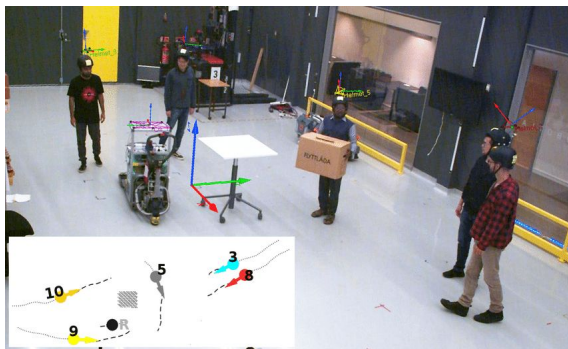
- v_{ij} : relative velocity between agent i and j
- d_{ij} : distance between agent i and j
- θ_{ij} : angle between agent i and j

Are all the observable variables useful to understand the observed scenario?



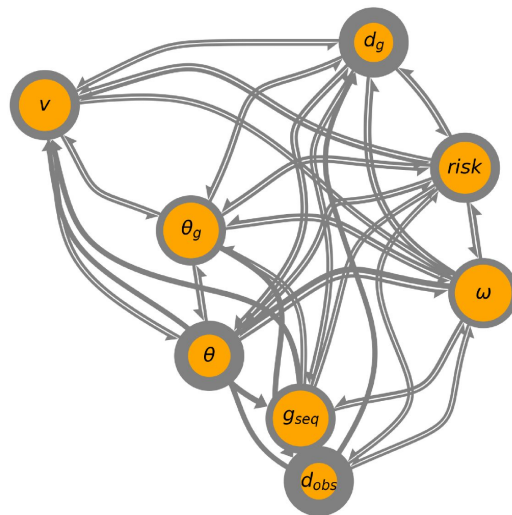
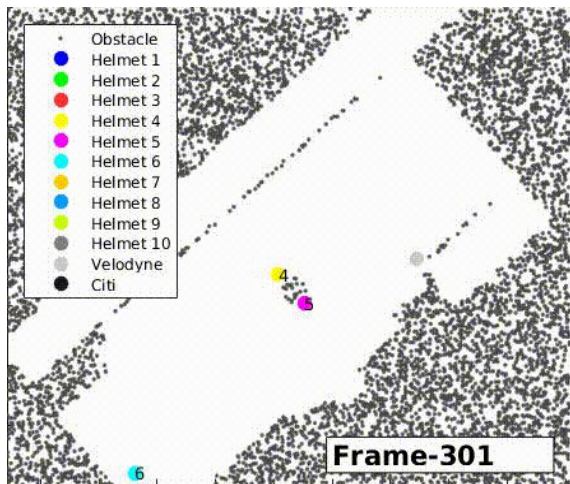
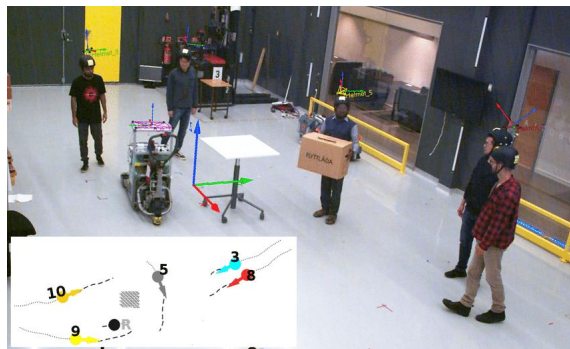
Robotics Applications

 F-PCMC algorithm

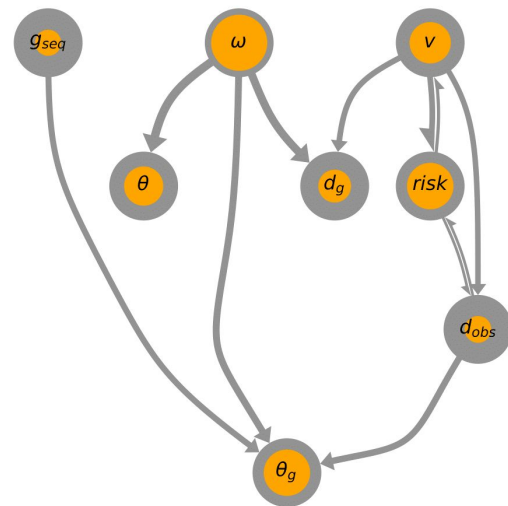


Robotics Applications

F-PCMCI algorithm



PCMCI ~80mins



F-PCMCI ~18mins

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- **Robotics Applications**



F-PCMCI algorithm



CAnDOIT algorithm



CausalFlow



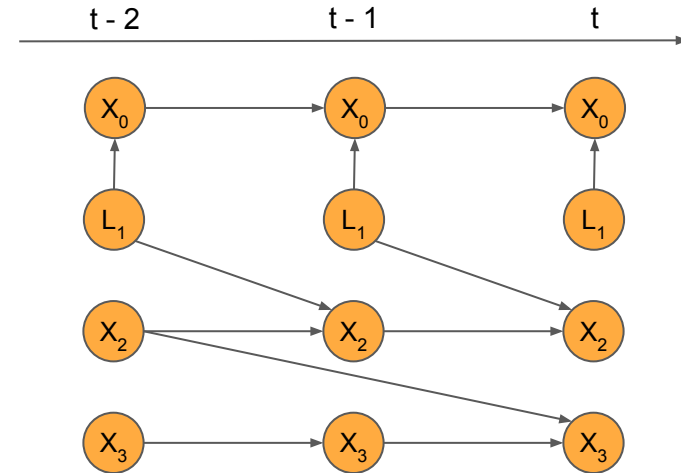
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Robotics Applications

CAnDOIT algorithm

- Observational data alone are often insufficient to accurately identify the correct causal model in complex scenarios where not all variables are observable

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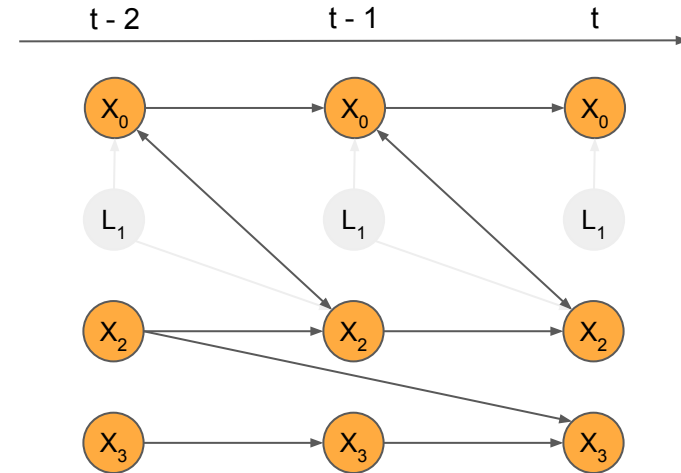


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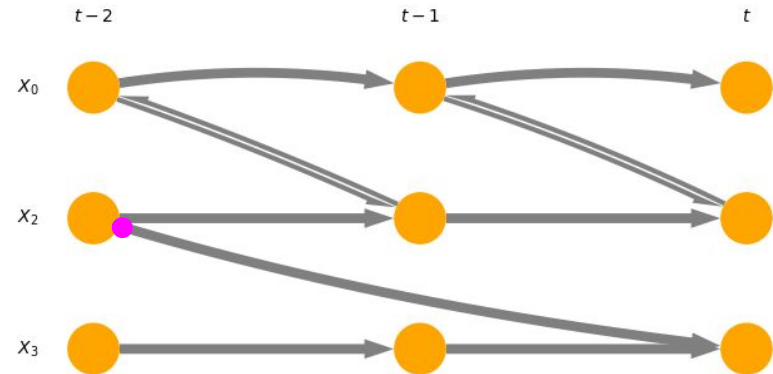
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Causal model obtained by using **Latent-PCMCI** (**LPCMCI**): version of PCMCI, based on FCI, that handles latent variables



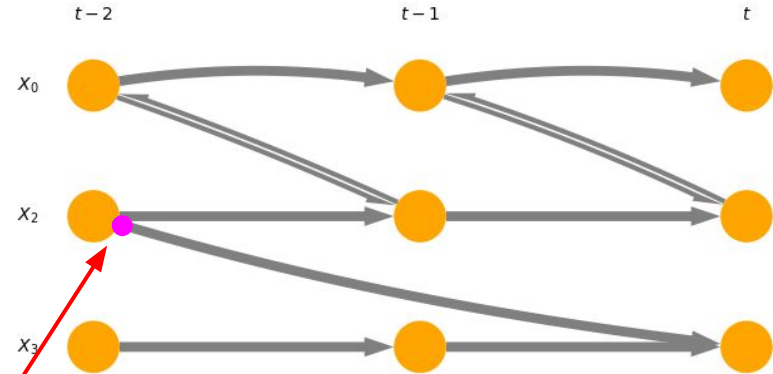
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Causal model obtained by using **Latent-PCMCI (LPCMCI)**: version of PCMCI, based on FCI, that handles latent variables



LPCMCI is uncertain about the orientation of this link

 = \rightarrow or \leftrightarrow

Robotics Applications

- Observational data alone are often insufficient to accurately identify the correct causal model in complex scenarios where not all variables are observable

Despite the "simple" toy problem (linear, 4 variables)



Causal model obtained by using **Latent-PCMCI (LPCMCI)**: version of PCMCI, based on FCI, that handles latent variables

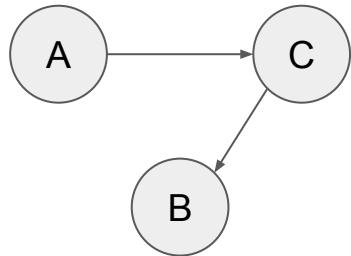
How can we perform causal discovery using data from observations 👁 and interventions 🛠 ?

Robotics Applications

CAnDOIT algorithm

- Observational data alone are often insufficient to accurately identify the correct causal model in complex scenarios where not all variables are observable
- We need **interventions**

CAnDOIT: CAusal Discovery with Observational  and Interventional  data from Time-series

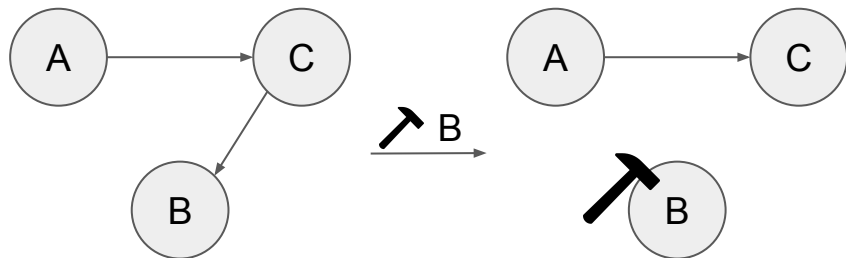


Robotics Applications

🔨 CAnDOIT algorithm

- Observational data alone are often insufficient to accurately identify the correct causal model in complex scenarios where not all variables are observable
- We need **interventions**

CAnDOIT: CAusal Discovery with Observational 👁️ and Interventional 🔨 data from Time-series



- For the observational case, we need to consider B's parents
- For the interventional case, we need to remove all incoming links to B

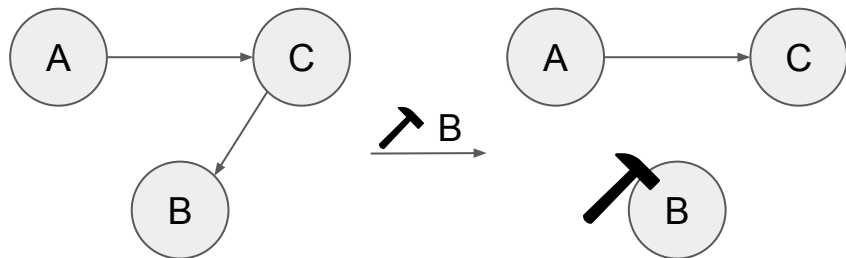
How can we enable a causal discovery method to do this?

Robotics Applications

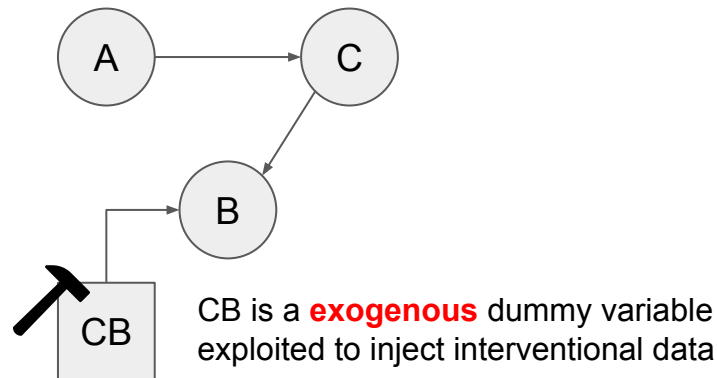
🔨 CAnDOIT algorithm

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CAnDOIT: CAusal Discovery with Observational 👁️ and Interventional 🔨 data from Time-series



🔨 CAnDOIT uses **context** variables

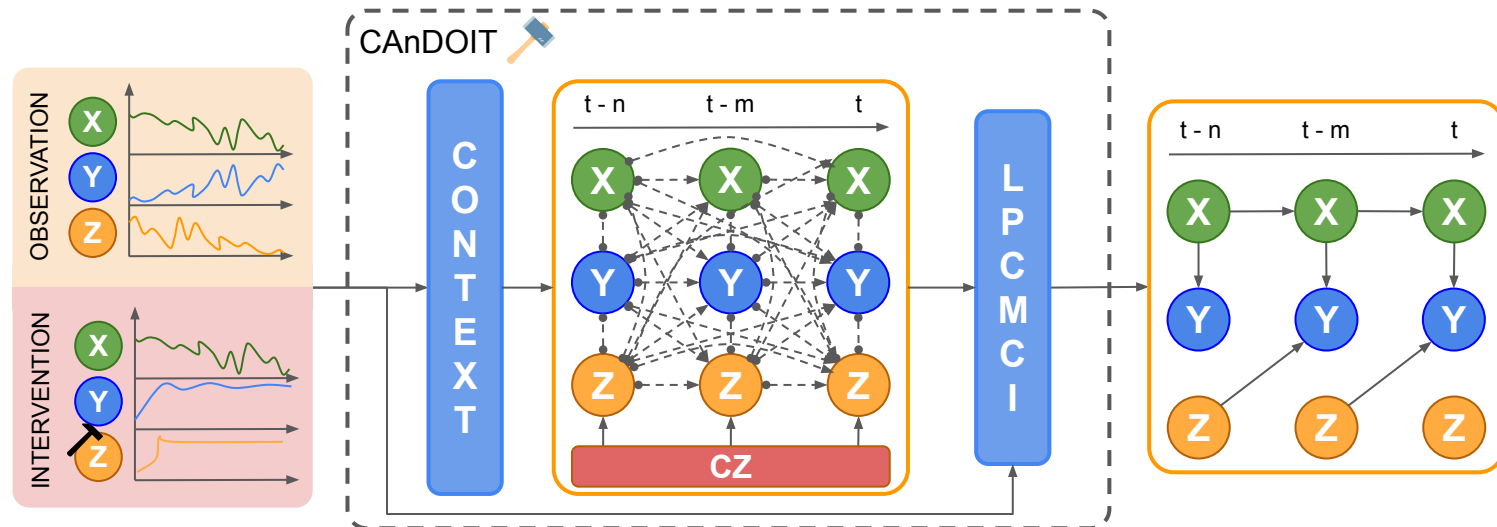


Robotics Applications

🔨 CAnDOIT algorithm

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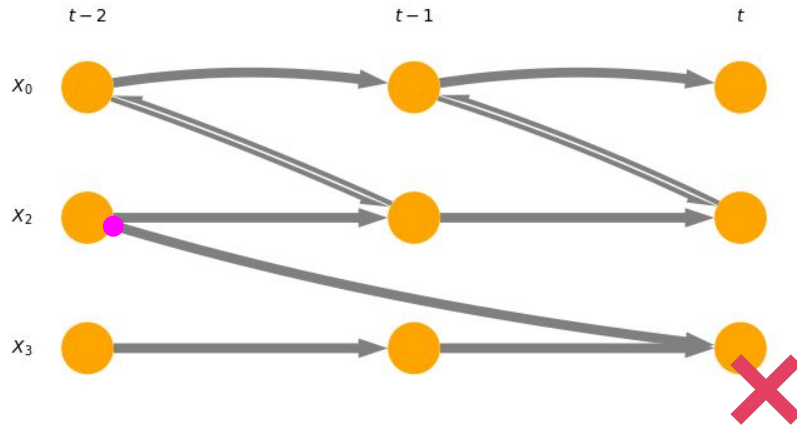
CAnDOIT: CAusal Discovery with Observational 👁️ and Interventional 🔨 data from Time-series



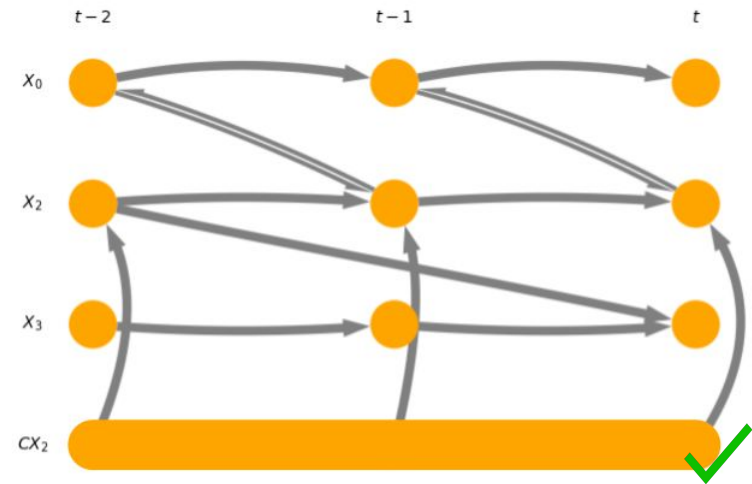
Robotics Applications

CAnDOIT algorithm

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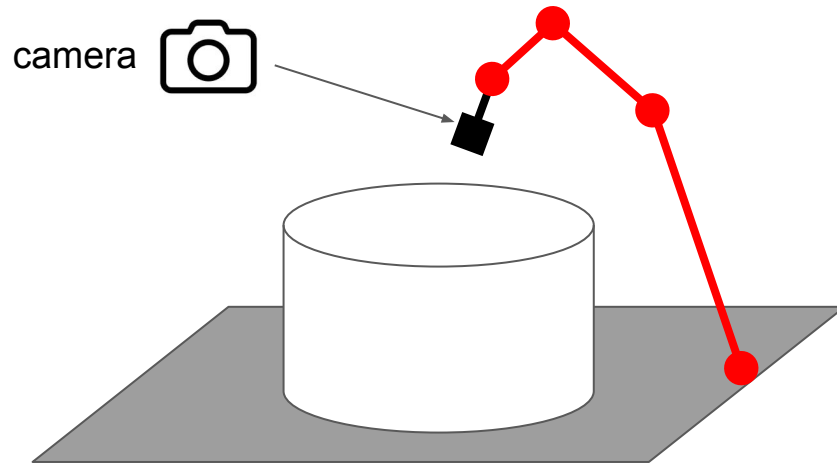
Causal model obtained by using **CAnDOIT**



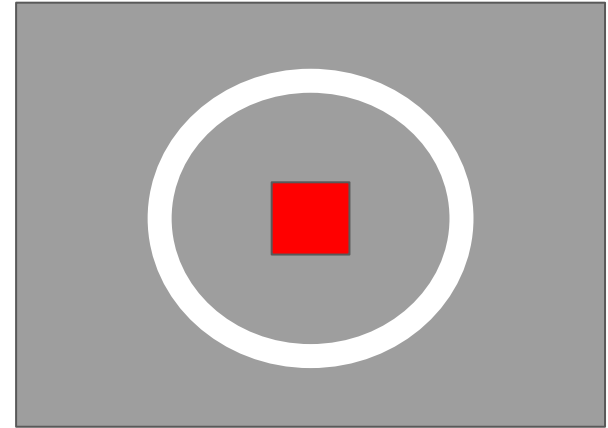
Robotics Applications

CAnDOIT algorithm

Consider the brightness of the colours of the objects in the cylinder captured by a robot camera



3D representation

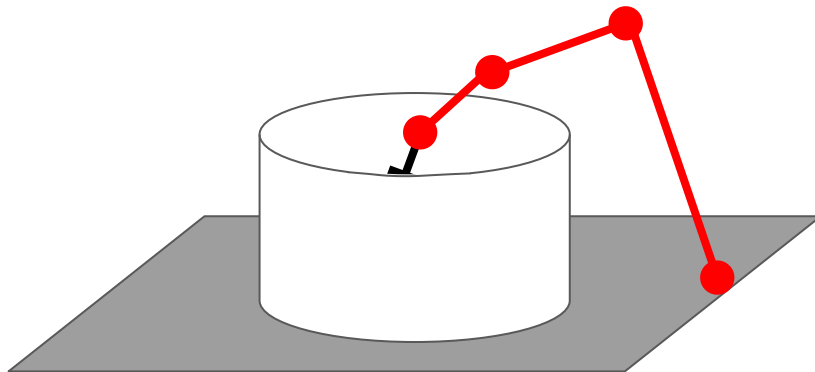


2D representation

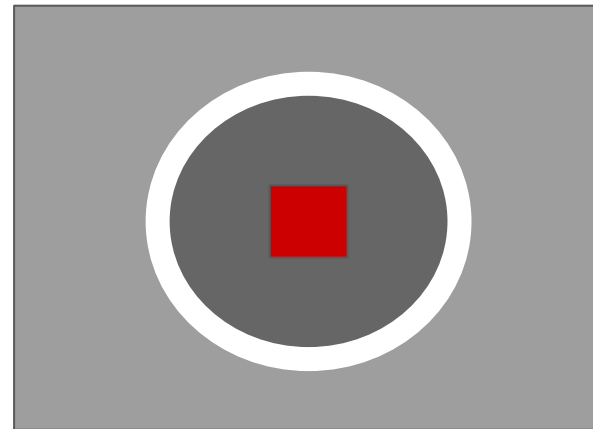
Robotics Applications

🔧 CAnDOIT algorithm

Consider the brightness of the colours of the objects in the cylinder captured by a robot camera



3D representation



2D representation

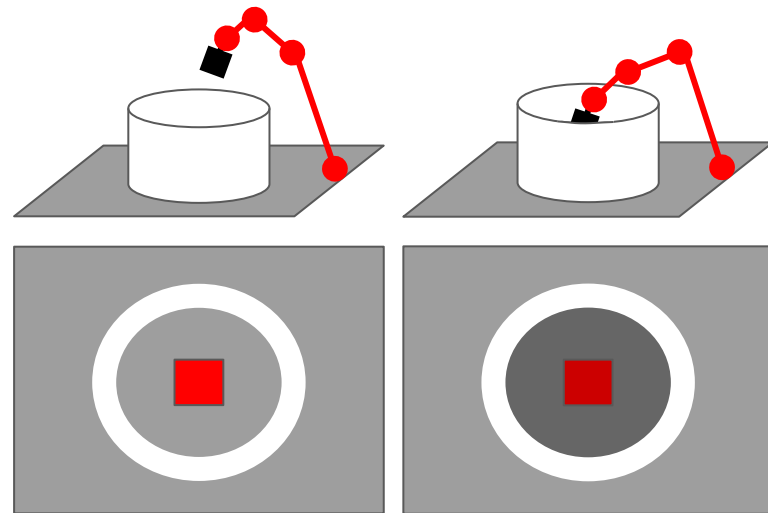
Robotics Applications

CAnDOIT algorithm

Consider the brightness of the colors of the objects in the cylinder captured by a robot camera

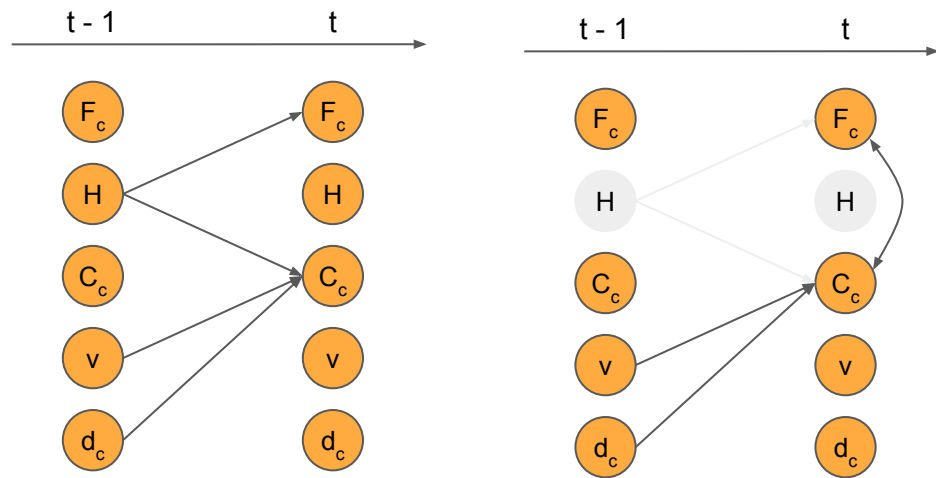
$$\begin{cases} F_c(t) = b(H(t-1)) \\ C_c(t) = b(H(t-1), v(t-1), d_c(t-1)) \end{cases}$$

$$b = K_h \frac{H}{H_{max}} + K_v \left(1 - \frac{v}{v_{max}}\right) + K_d \frac{d_c}{d_{cmax}}$$

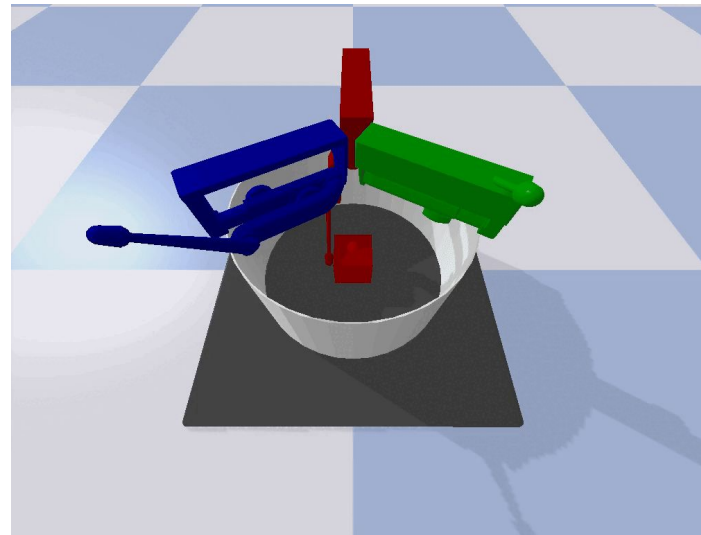


Robotics Applications

🔧 CAnDOIT algorithm

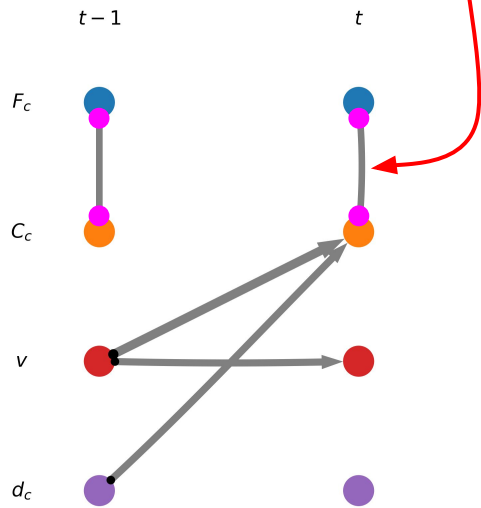
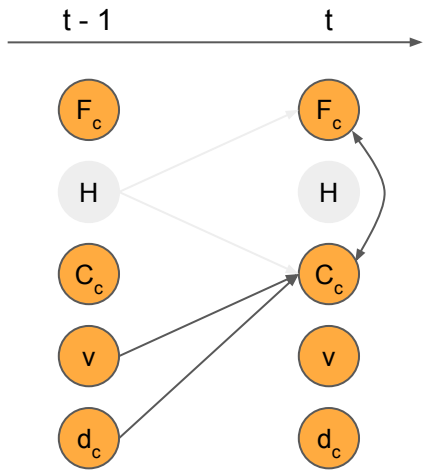


$$\begin{cases} F_c(t) = b(H(t-1)) \\ C_c(t) = b(H(t-1), v(t-1), d_c(t-1)) \end{cases}$$

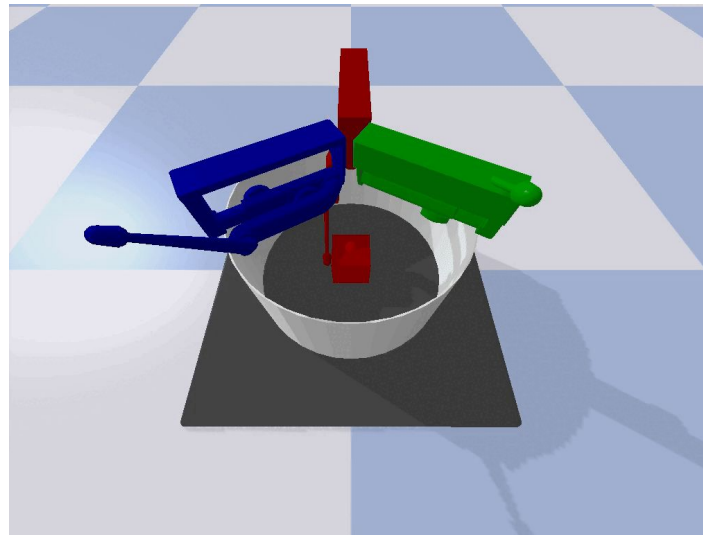


Robotics Applications

🔧 CAnDOIT algorithm



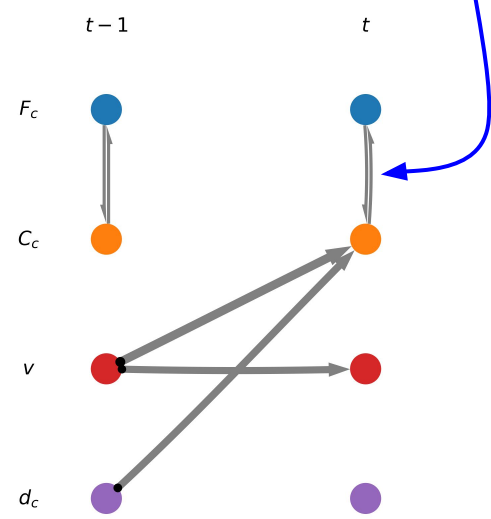
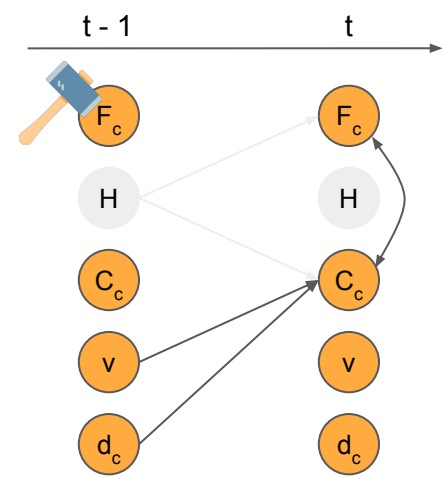
⬅️ ? ➡️
🧑
Again, LPCMCI is uncertain about the orientation of this link



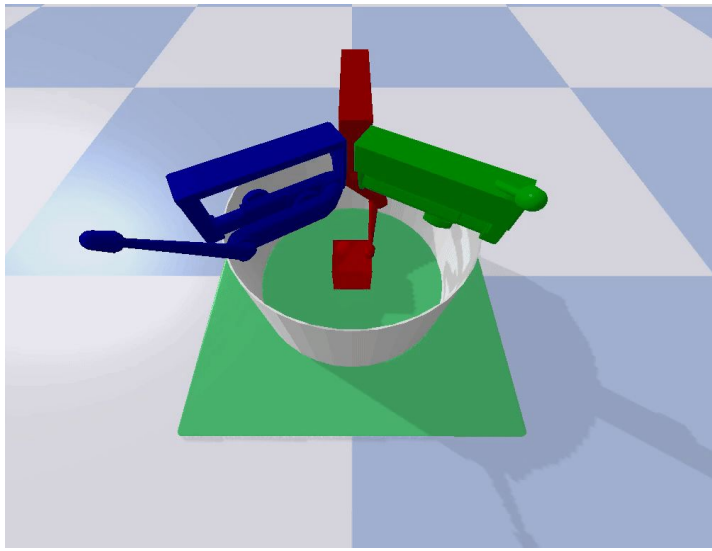
$$\begin{cases} F_c(t) = b(H(t-1)) \\ C_c(t) = b(H(t-1), v(t-1), d_c(t-1)) \end{cases}$$

Robotics Applications

 CAnDOIT algorithm



CAnDOIT using observational and interventional data is able to correctly orient this link



$$\begin{cases} F_c(t) = b(\cancel{H(t-1)}) \text{ } \bullet \\ C_c(t) = b(H(t-1), v(t-1), d_c(t-1)) \end{cases}$$

Outline

- Causal Discovery for Time-series Data
 - PCMCI algorithm

- **Robotics Applications**



F-PCMCI algorithm



CAnDOIT algorithm



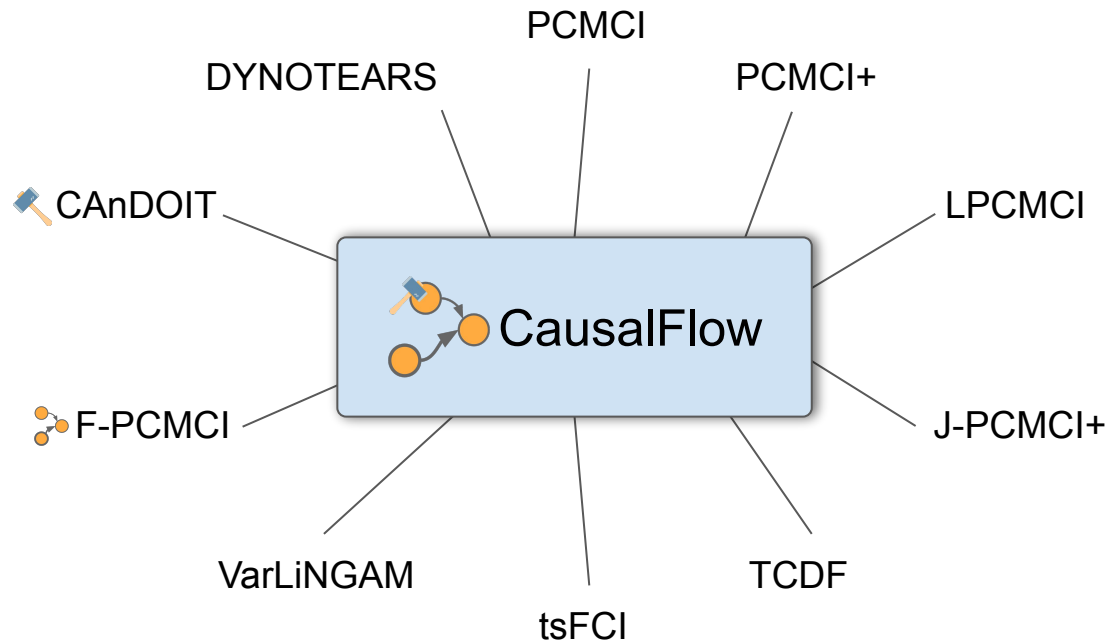
CausalFlow



ROS-Causal

Robotics Applications

 CausalFlow



GitHub



 CausalFlow

```
pip install  
py-causalflow
```

Outline

- Causal Discovery for Time-series Data
 - PCMCI algorithm

- **Robotics Applications**



F-PCMCI algorithm



CAnDOIT algorithm



CausalFlow



ROS-Causal

What is Robot Operating System (ROS)?



people tracker

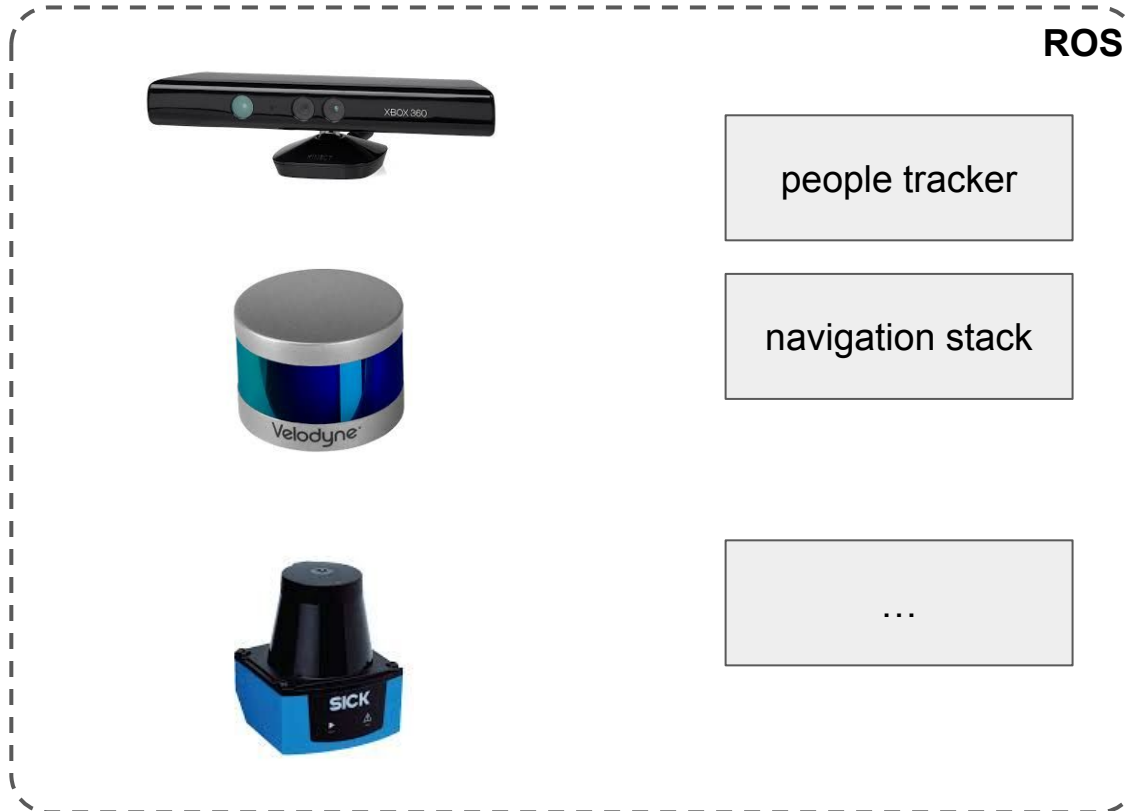


navigation stack

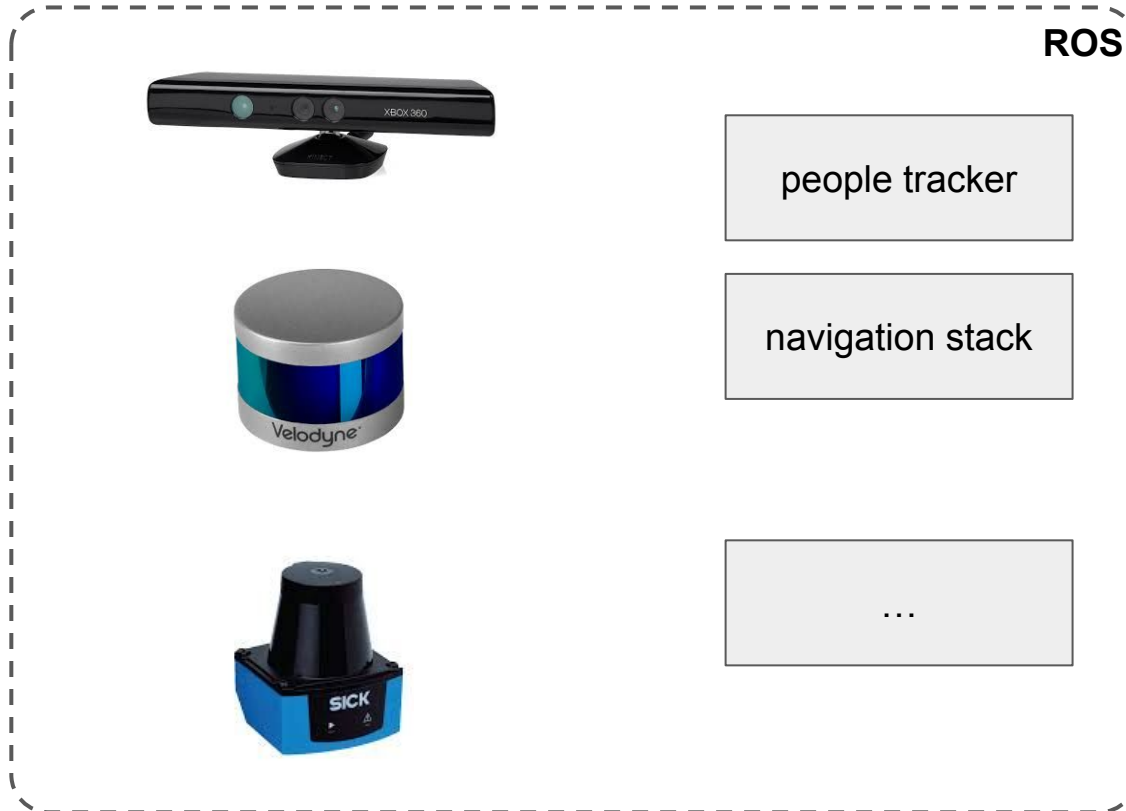


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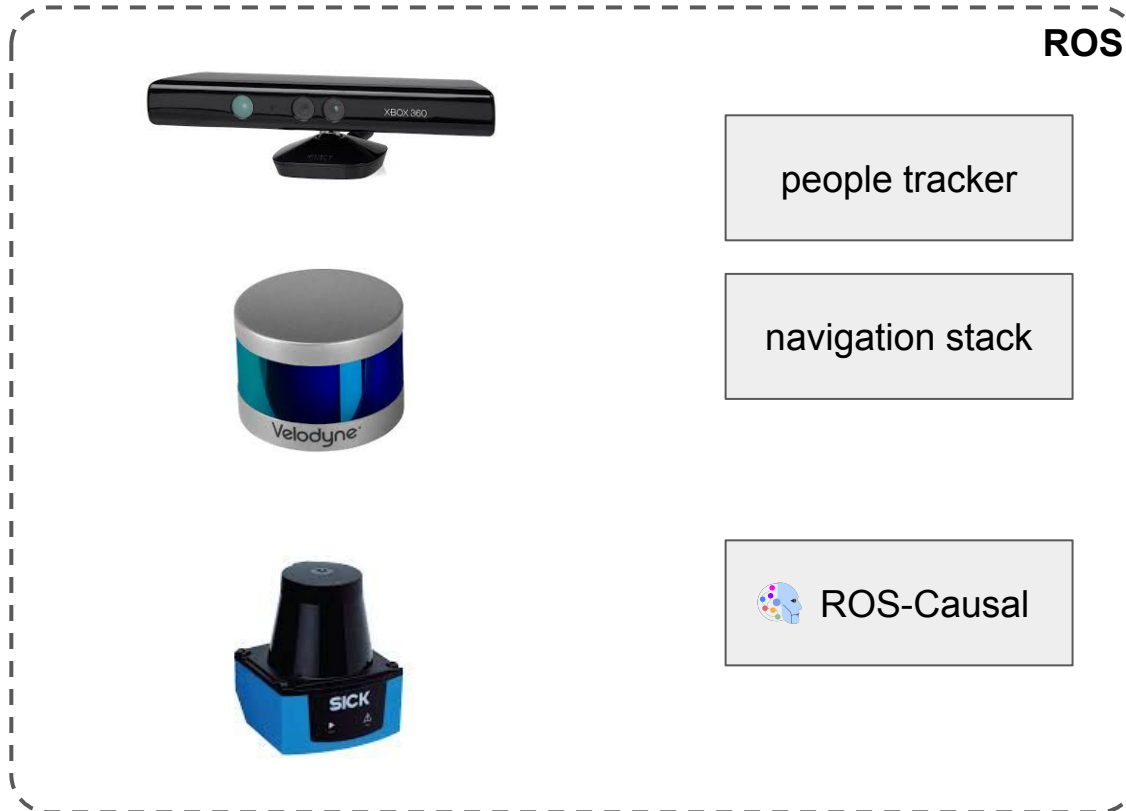
What is Robot Operating System (ROS)?



What is Robot Operating System (ROS)?

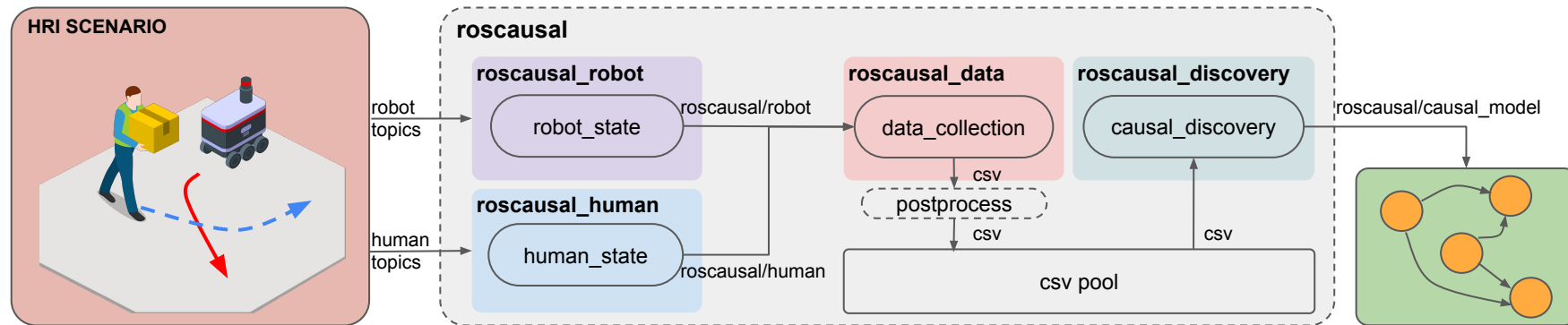


What is Robot Operating System (ROS)?



Robotics Applications

ROS-Causal

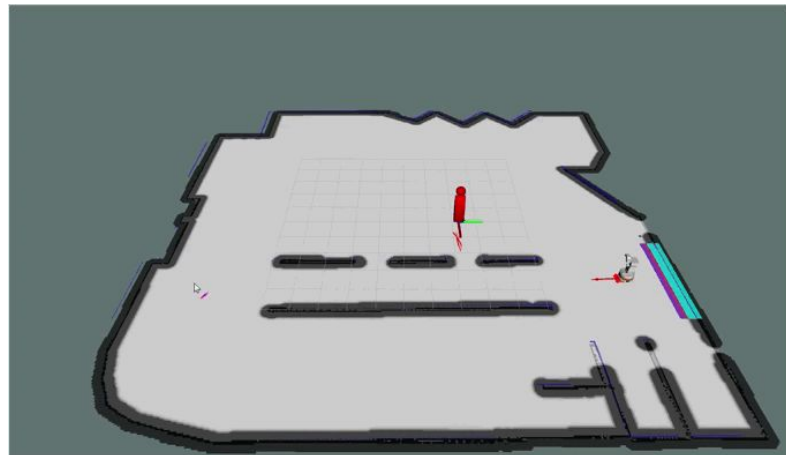
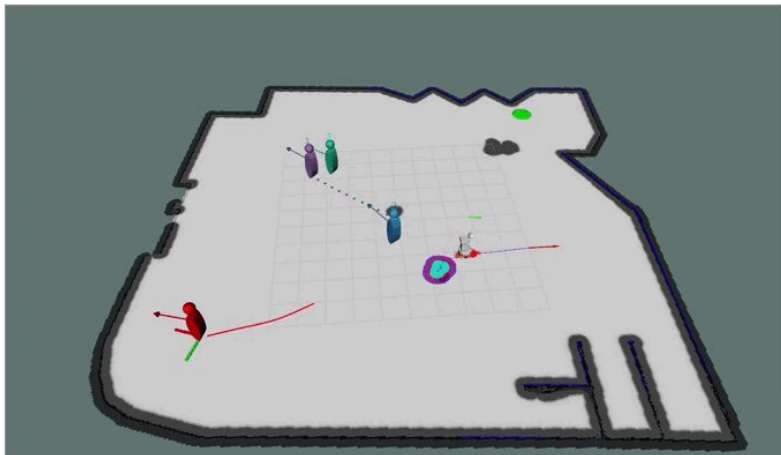
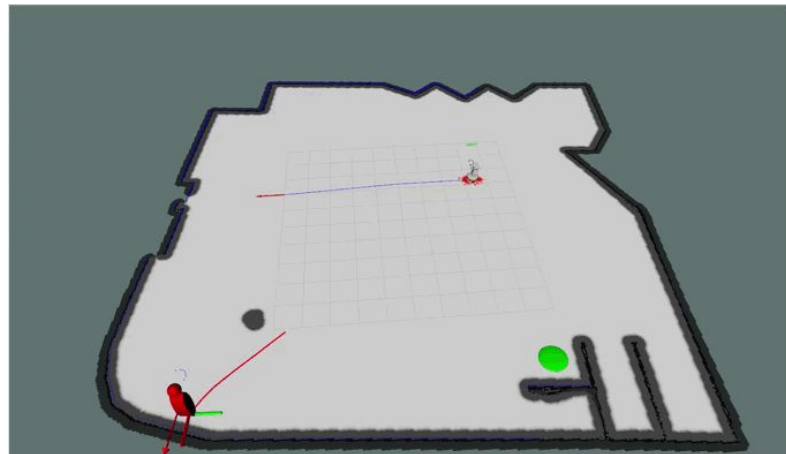


- ROS-Causal extracts and collects data from a HRI scenario, such as agents' trajectories, and performs causal analysis on the collected data in a batched manner. It is composed by four different rosnodes:
 - **roscasual_robot**
 - **roscasual_human**
 - **roscasual_data**
 - **roscasual_discovery**

ROS-Causal_HRISim

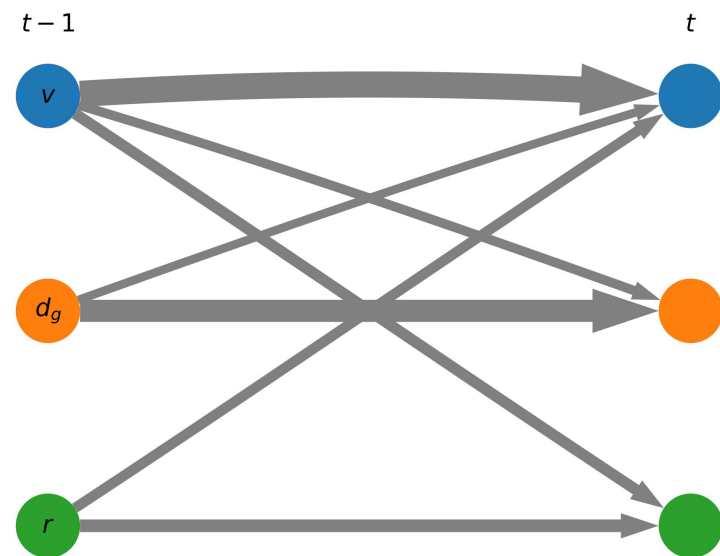
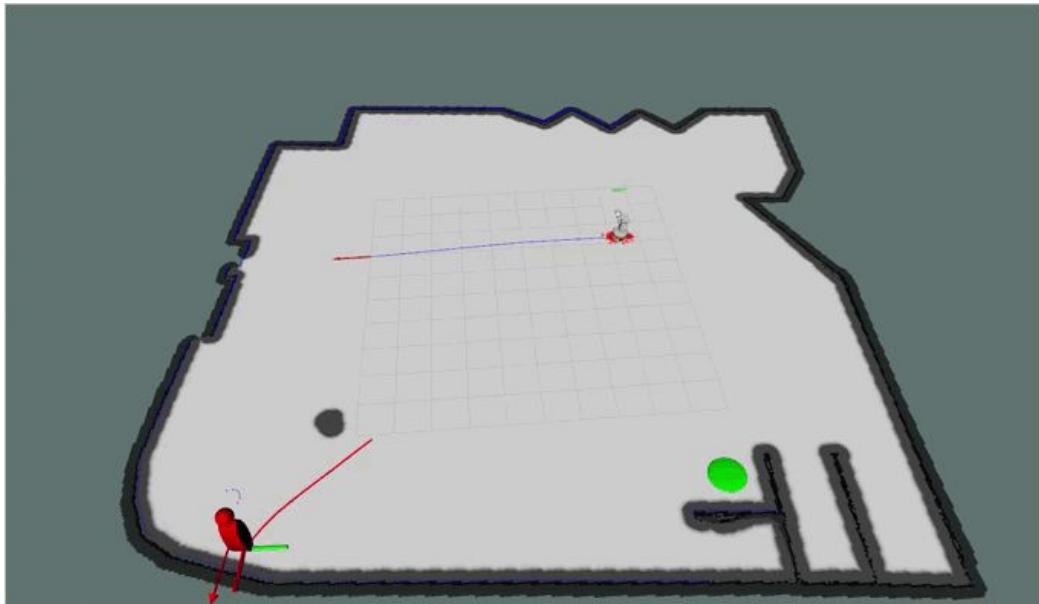
HRI simulator involving:

- TIAGo robot
- pedestrians



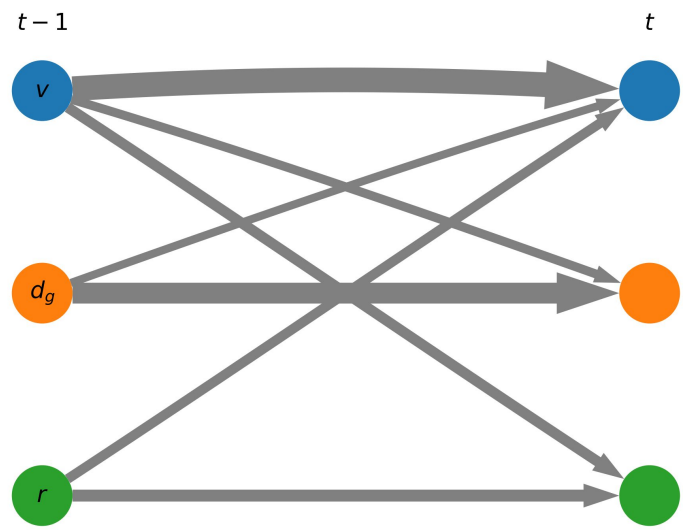
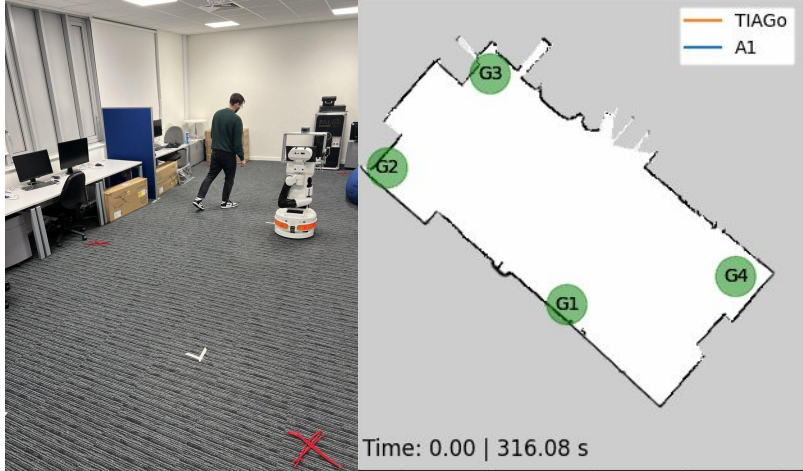
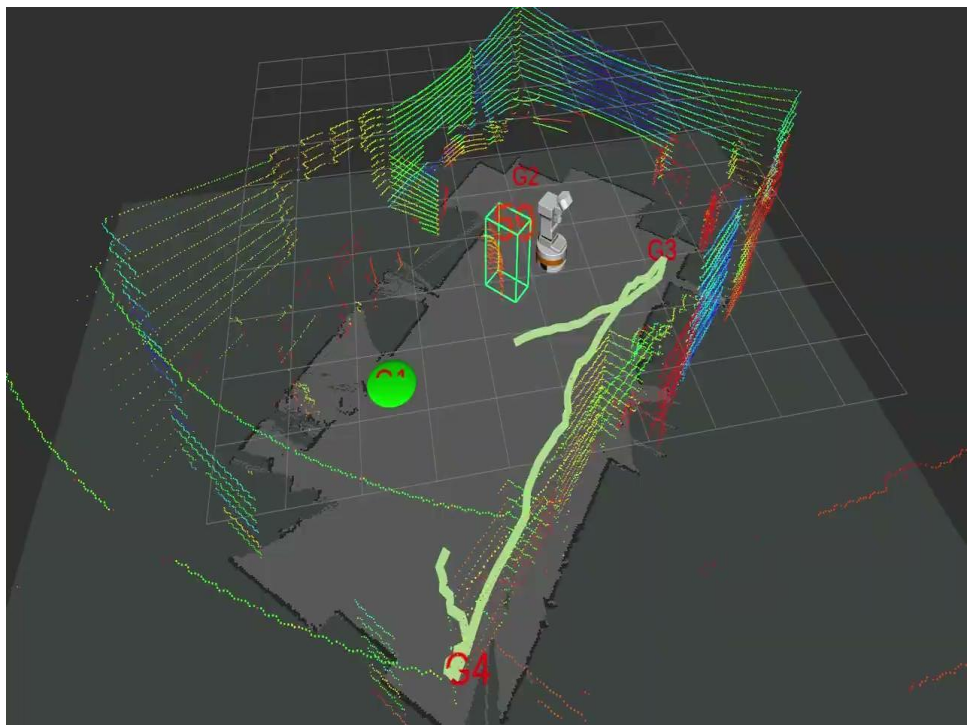
Robotics Applications

 ROS-Causal



Robotics Applications

ROS-Causal



Reference

- Runge, J., Nowack, P., Kretschmer, M., Flaxman, S. and Sejdinovic, D., 2019. Detecting and quantifying causal associations in large nonlinear time series datasets. Science advances, 5(11).
- Castri, L., Mghames, S., Hanheide, M. and Bellotto, N., 2023. Enhancing Causal Discovery from Robot Sensor Data in Dynamic Scenarios. In 2nd Conference on Causal Learning and Reasoning.
- Castri, L., Beraldo, G., Mghames, S., Hanheide M. and Bellotto, N. 2024. Experimental Evaluation of ROS-Causal in Real-World Human-Robot Spatial Interaction Scenarios. In IEEE International Conference on Robot and Human Interactive Communication (RO-MAN).
- Castri, L., Mghames, S., Hanheide M. and Bellotto, N. 2024. CAnDOIT: Causal Discovery with Observational and Interventional Data from Time-Series, Advanced Intelligent Systems.



Thank you!
questions?