CausalFlow

A Unified Framework for Causality in Time-Series

UNIVERSITY OF LINCOLN UNITED KINGDOM

Luca Castri
PhD student
University of Lincoln

Website: ***https://darko-project.eu***
This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 101017274

DARKO
Dynamic Agile Production Robots That Learn and Optimise Knowledge and Operations

Outline

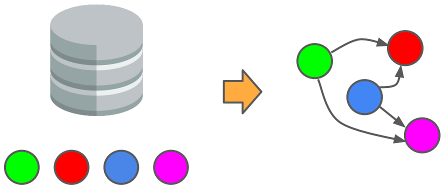
- Motivation
- CausalFlow
- F-PCMCI
- CAnDOIT
- RandomGraph
- Conclusion & Future directions

Motivation

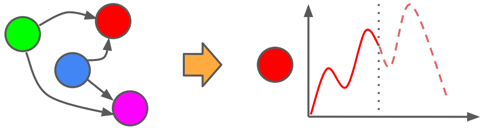
What is causality?

> “Science that studies the cause-and-effect relationship between events”
> [Pearl, J., & Mackenzie, D. (2019). The book of why]

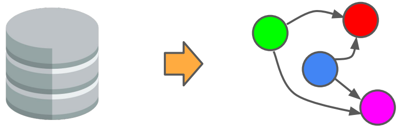
Causal Structure Learning



Causal Reasoning



Causal Representation Learning



Motivation

- Many causal discovery methods

- Different input format
- Different output format
- Different graphs

→ A <u>unified</u> framework is needed to handle diverse inputs, outputs, and graph types in causal discovery

input ➡	method ➡	output ➡	graph
numpy array	PCMCI	numpy matrix	networkX
dataframe	DYNOTEARS	dictionary	custom
custom	VarLiNGAM	custom	...
...	tsFCI	...	
	TCDF		
	...		

CausalFlow

A Unified Framework for Causality in Time-Series

A suite of causal discovery methods from time-series

- CAnDOIT
- F-PCMCI
- PCMCI
- PCMCI+
- LPCMCI
- J-PCMCI+
- TCDF
- tsFCI
- DYNOTEARS
- VarLiNGAM

RandomGraph

- random systems of equations with(out) hidden confounders
- observational and interventional data from the generated graph
- various adjustable parameters (time-series length, obs vars, hidden vars, etc..)

CausalFlow

A Unified Framework for Causality in Time-Series

numpy array → dataframe → CSV path → Data object → discovery → Graph object → Summary graph / Time-series graph

Data object

- Properties:
  - N
  - T
  - varname
  - dataframe
- Methods
  - plot
  - fill nan
  - subsampling

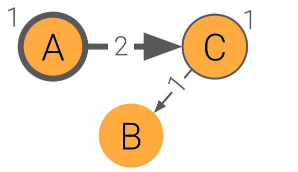
discovery

- CAnDOIT
- F-PCMCI
- PCMCI
- PCMCI+
- LPCMCI
- J-PCMCI+
- TCDF
- tsFCI
- DYNOTEARS
- VarLiNGAM

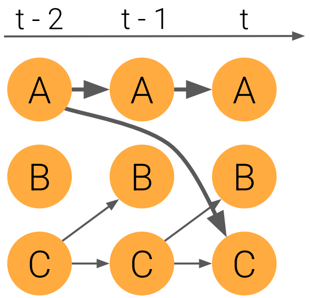
Graph object

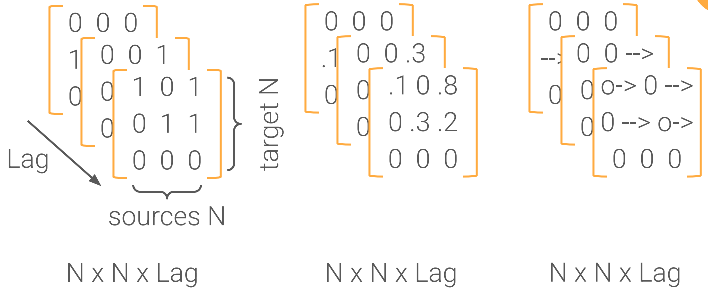
- 3D Matrices
  - Skeleton
  - Value
  - P-value
  - Link

Summary graph



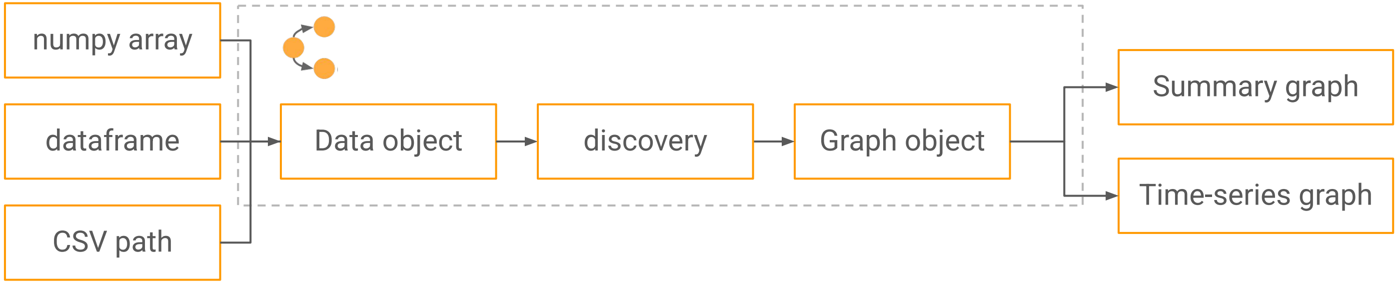
Time-series graph

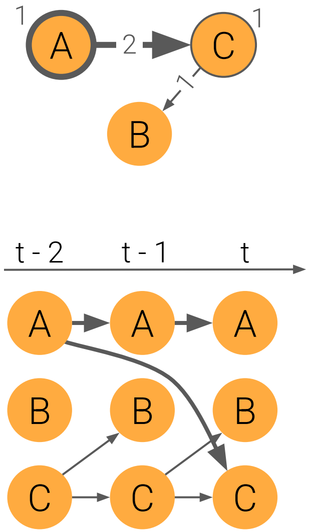




CausalFlow

A Unified Framework for Causality in Time-Series





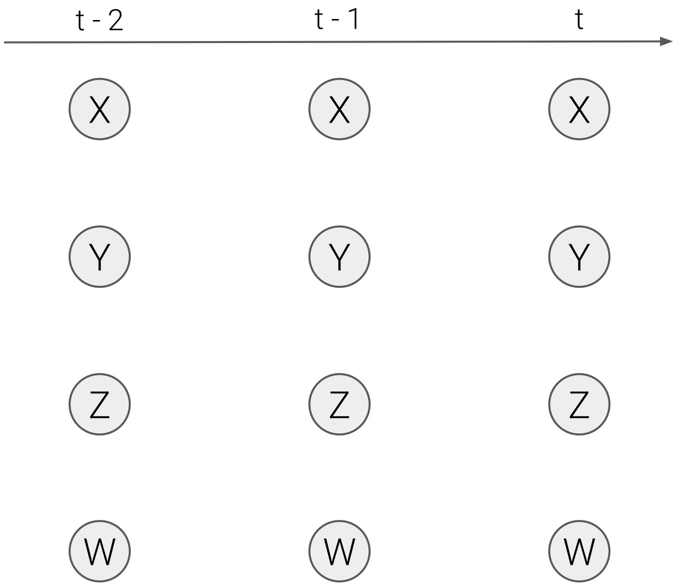
F-PCMCI

Fast and accurate causal discovery algorithm for time-series

PCMCI [Runge et al. 2019]

- PC algorithm
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables
- MCI test
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t | \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$



F-PCMCI

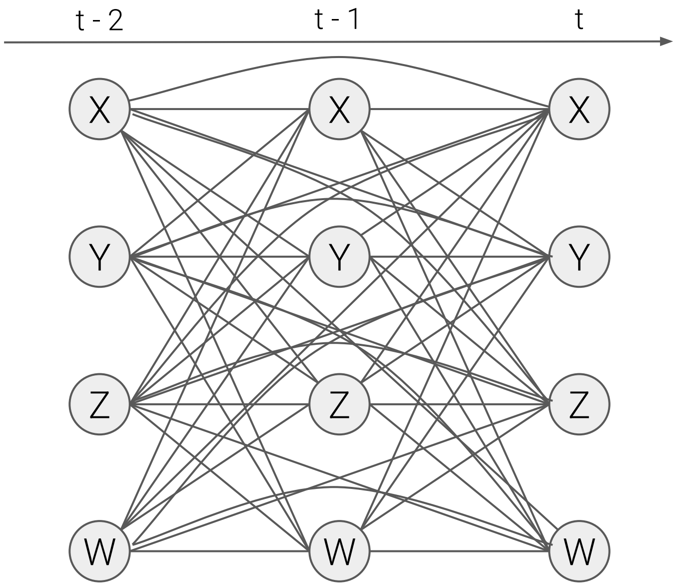
Fast and accurate causal discovery algorithm for time-series

PCMCI [Runge et al. 2019]

- PC algorithm
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- MCI test
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t | \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$



F-PCMCI

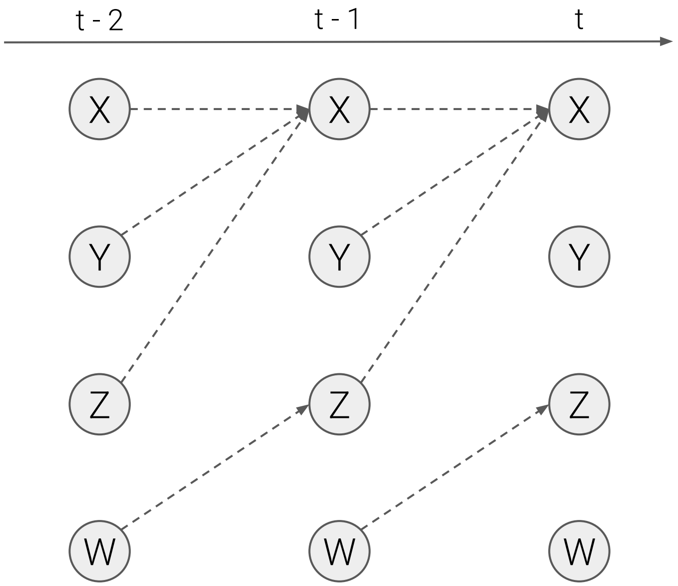
Fast and accurate causal discovery algorithm for time-series

PCMCI [Runge et al. 2019]

- PC algorithm
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- MCI test
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$X^i_{t-\tau} \perp\!\!\!\perp X^j_t \,|\, \tilde{P}(X^i_{t-\tau}), \tilde{P}(X^j_t)$$



F-PCMCI

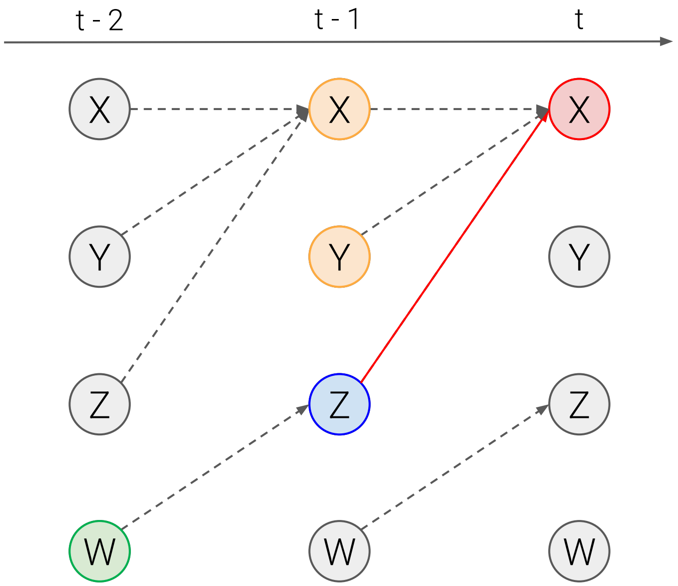
Fast and accurate causal discovery algorithm for time-series

PCMCI [Runge et al. 2019]

- PC algorithm
  retrieves the causal model structure by considering ONLY lagged dependencies as possible causal relationships between variables

- MCI test
  validates the structure found at the previous step by performing a false positive rate optimisation control

$$\boxed{X^i_{t-\tau}} \perp\!\!\!\perp \boxed{X^j_t} \,|\, \boxed{\tilde{P}(X^i_{t-\tau})}, \boxed{\tilde{P}(X^j_t)}$$



F-PCMCI

Fast and accurate causal discovery algorithm for time-series

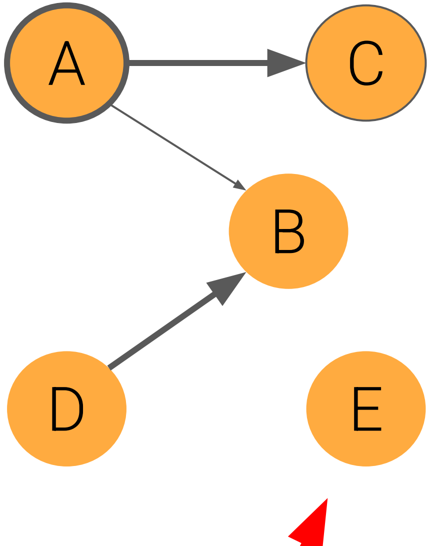
- PCMCI computational complexity

$$\mathcal{O}(N^3\tau_{\max}^2 + N^2\tau_{\max})$$

- Is it possible to improve the causal discovery process?
- Are all observed variables useful?

GOAL

- Build an all-in-one solution to select key variables and reconstruct a causal model



Is node E essential to understand the evolution of the observed system?

F-PCMCI

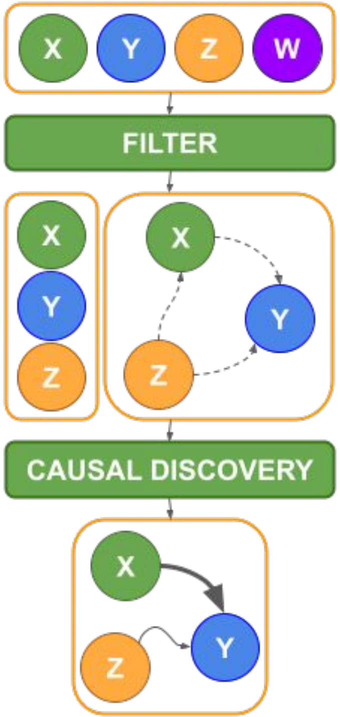
Fast and accurate causal discovery algorithm for time-series

Is it possible to improve the causal discovery process?

Filtered-PCMCI (F-PCMCI)

1. predefined set of variables
2. remove irrelevant variables using transfer entropy
3. build hypothetical causal structure from reduced set
4. run PCMCI on hypothetical model

→ **Faster** and **more accurate** causal discovery



F-PCMCI

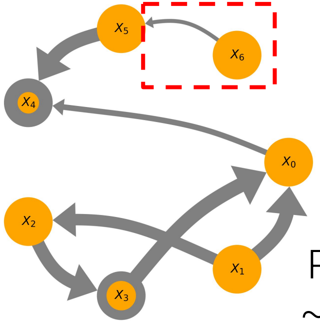
Fast and accurate causal discovery algorithm for time-series

Is it possible to improve the causal discovery process?

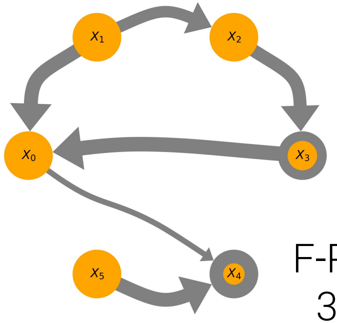
Toy problem

$$\begin{cases} x_0(t) = 2x_1(t-1) + 3x_3(t-1) + \eta_0 \\ x_1(t) = \eta_1 \\ x_2(t) = 1.1x_1(t-1)^2 + \eta_2 \\ x_3(t) = x_3(t-1) \cdot x_2(t-1) + \eta_3 \\ x_4(t) = x_4(t-1) + x_5(t-1) \cdot x_0(t-1) \\ x_5(t) = \eta_5 \\ x_6(t) = \eta_6 \end{cases}$$

isolated

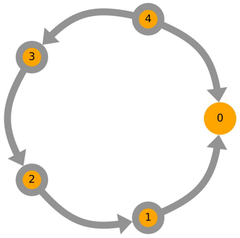


PCMCI
~9mins

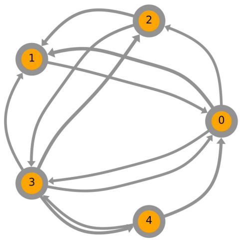


F-PCMCI
3mins

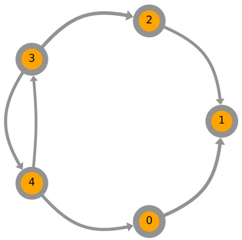
fMRI data [Smith et al. 2011]



GT



PCMCI



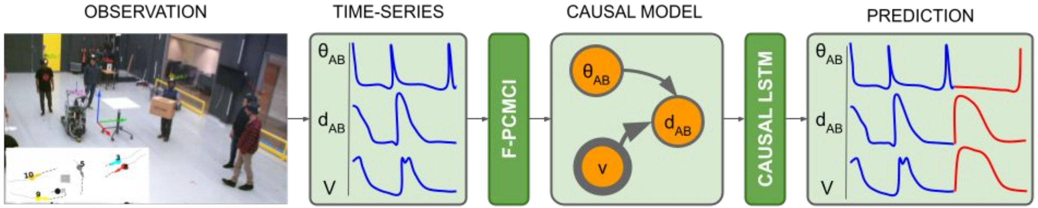
F-PCMCI

	SHD	F1-Score	Time
PCMCI	8	0.69	90'50”
F-PCMCI	4	0.80	38'52”

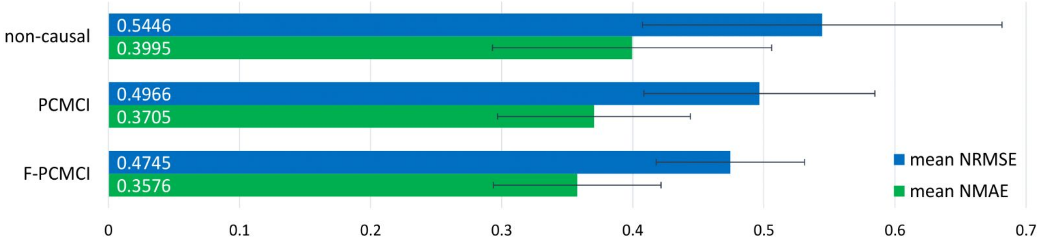
F-PCMCI

Fast and accurate causal discovery algorithm for time-series

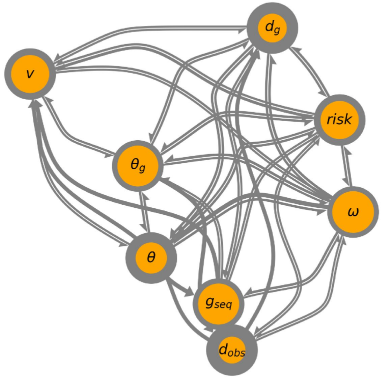
Is it possible to improve the causal discovery process?



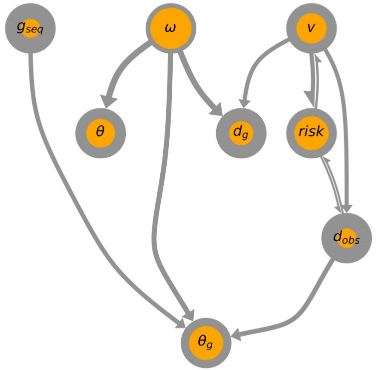
- No ground-truth causal model
- Prediction accuracy used to evaluate causal models



PCMCI ~80mins



F-PCMCI ~18mins



F-PCMCI

Fast and accurate causal discovery algorithm for time-series

Summing up

- F-PCMCI for fast and accurate causal discovery

Research outcomes

- Castri et al. "**Enhancing causal discovery from robot sensor data in dynamic scenarios**," Conference on Causal Learning and Reasoning, 2023.

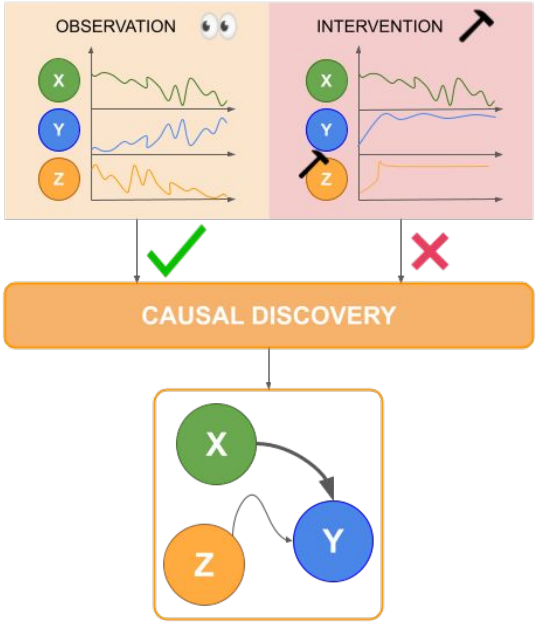
Main limitation

- Time-series causal discovery uses only observations. Can interventions help?

🔨 CAnDOIT

Observation and intervention-based causal discovery algorithm for time-series

- Observational data alone are often insufficient to identify the correct causal model
- Time-series methods do not integrate interventional data
- Can causal discovery integrate observational and interventional time-series?



GOAL

- First causal discovery method for time-series that uses both observational and interventional data

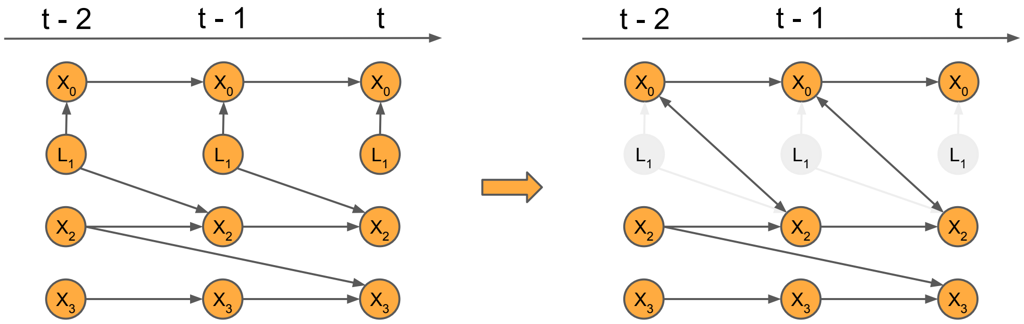
CAnDOIT

Observation and intervention-based causal discovery algorithm for time-series

Can causal discovery integrate observational and interventional time-series?

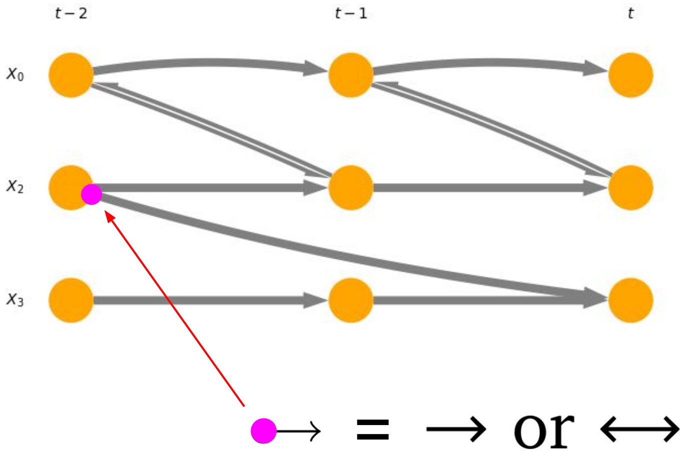
$$\begin{cases} X_0(t) = 0.9X_0(t-1) + 0.6X_1(t) + \eta_0 \\ L_1(t) = \eta_1 \\ X_2(t) = 0.9X_2(t-1) + 0.4X_1(t-1) + \eta_2 \\ X_3(t) = 0.9X_3(t-1) - 0.5X_2(t-2) + \eta_3 \end{cases}$$

LATENT



LPCMCI [Gerhardus et al. 2020]

- based on FCI
- handles latent confounders



🔨 CAnDOIT

Observation and intervention-based causal discovery algorithm for time-series

Can causal discovery integrate observational and interventional time-series?

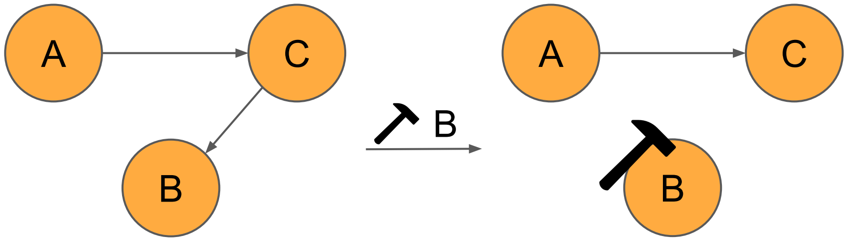
🔨 CAnDOIT

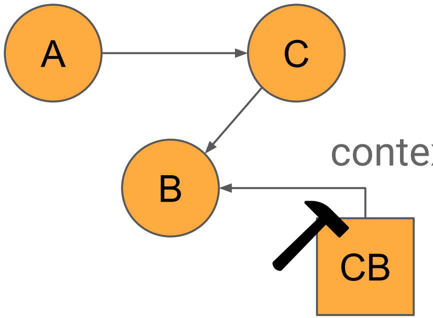
CAusal **D**iscovery with **O**bservational 👀 and **I**nterventional 🔨 data from **T**ime-series

HARD INTERVENTION

- observation: use B’s parents
- intervention: remove all inputs to B

How to build this into causal discovery?





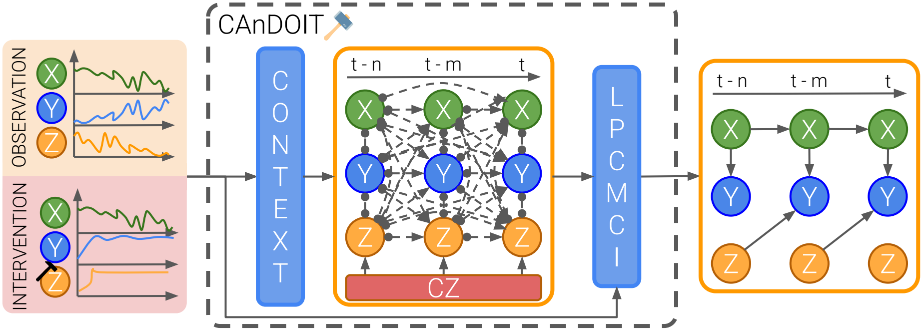
context variables to inject interventional data
[Mooij et al. 2020]

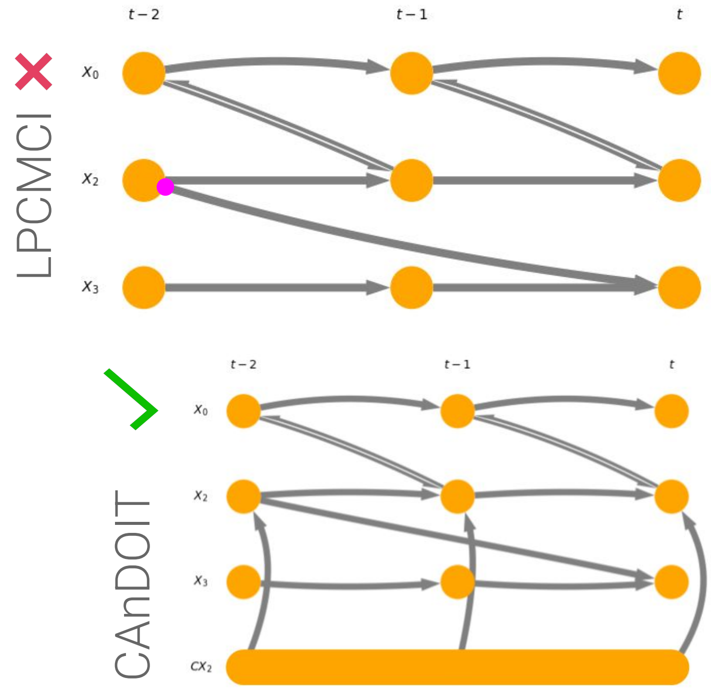
🔨 CAnDOIT

Observation and intervention-based causal discovery algorithm for time-series

Can causal discovery integrate observational and interventional time-series?

$$
\begin{cases}
X_0(t) = 0.9X_0(t-1) + 0.6X_1(t) + \eta_0 \\
L_1(t) = \eta_1 \\
X_2(t) = 0.9X_2(t-1) + 0.4X_1(t-1) + \eta_2 \\
X_3(t) = 0.9X_3(t-1) - 0.5X_2(t-2) + \eta_3
\end{cases}
$$





🔨 CAnDOIT

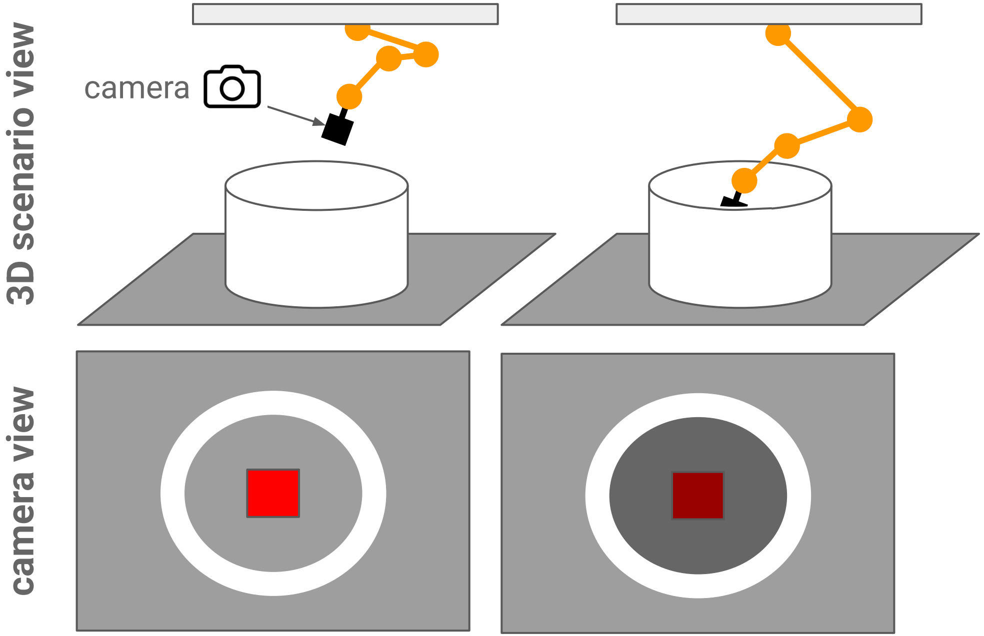
Observation and intervention-based causal discovery algorithm for time-series

Can causal discovery integrate observational and interventional time-series?

$$\begin{cases} F_c(t) = \boxed{b(H(t-1))} \\ C_c(t) = \boxed{b(H(t-1),}\, v(t-1), d_c(t-1)) \end{cases}$$

$$b = K_h \frac{H}{H_{max}} + K_v \left(1 - \frac{v}{v_{max}}\right) + K_d \frac{d_c}{d_{c_{max}}}$$

- Floor and cube colours' brightness influenced by:
    - camera height
    - camera velocity
    - camera distance to the cube

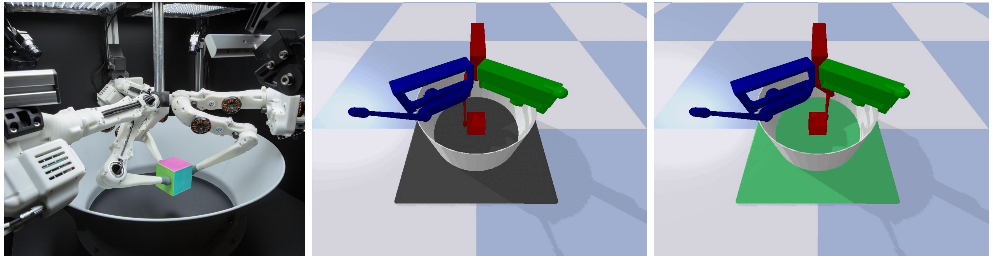


CAnDOIT

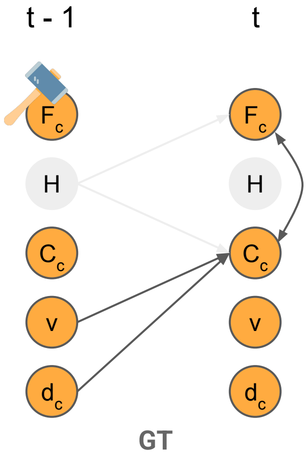
Observation and intervention-based causal discovery algorithm for time-series

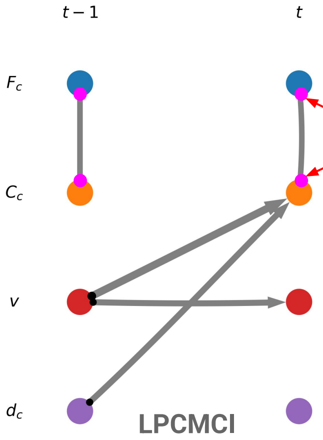
Can causal discovery integrate observational and interventional time-series?

$$\begin{cases} F_c(t) = \cancel{b(H(t-1))} \\ C_c(t) = b(H(t-1), v(t-1), d_c(t-1)) \end{cases}$$



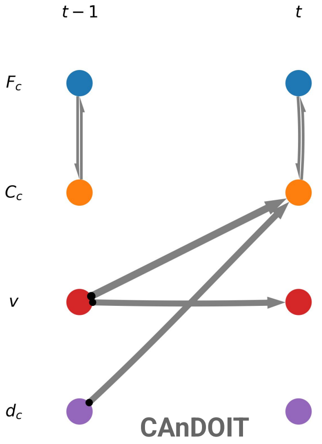
CausalWorld
[Ahmed et al. 2021]





uncertainties

●—● = → or ←



🔨 CAnDOIT

Observation and intervention-based causal discovery algorithm for time-series

Summing up

- ✅ First observation and intervention-based causal discovery method from time-series

Research outcomes

- Castri et al. "**CAnDOIT: Causal Discovery with Observational and Interventional Data from Time-Series**", Advanced Intelligent Systems, 2024.

RandomGraph

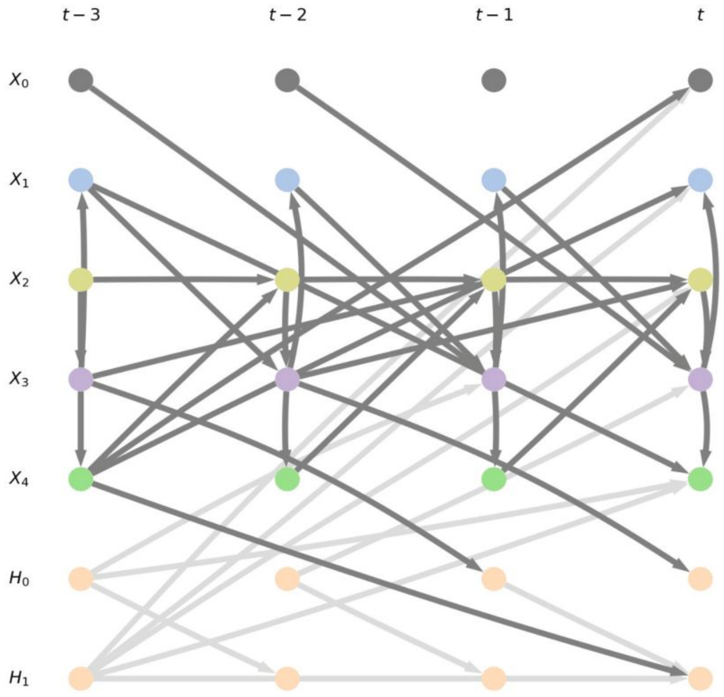
Generate random causal models for testing and benchmarking

- time-series length
- number of observable variables;
- number of observable parents per variable (link density)
- number of hidden confounders
- number of confounded variables per hidden confounder
- noise configuration [uniform, gaussian, weibull]
- minimum τ_{min} and maximum τ_{max} time delay
- coefficient range
- functional forms [–, sin, cos, abs, pow, exp], where – stands for none
- operators [$+, -, *, /$]

RandomGraph

```python
noise_gaussian = (NoiseType.Gaussian, 0, 1)
RS = RandomGraph(nvars = 5,
                 nsamples = 1500,
                 link_density = 3,
                 coeff_range = (0.1, 0.5),
                 max_exp = 2,
                 min_lag = 0,
                 max_lag = 3,
                 noise_config = noise_gaussian,
                 functions = ['','sin', 'cos', 'exp', 'abs', 'pow'],
                 operators = ['+', '-', '*', '/'],
                 n_hidden_confounders = 2)
RS.gen_equations()
```

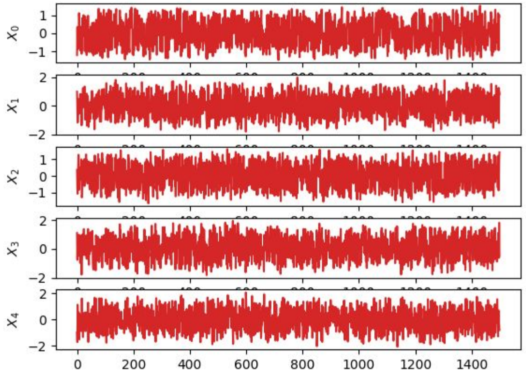
$$
\begin{cases}
X_0(t) = \dfrac{0.48\cos(X_4(t-3))}{0.12\sin(H_1(t-3))} \\
X_1(t) = 0.17\sin(X_4(t-3)) - 0.46\cos(X_3(t)) + 0.14|H_1|(t-3) \\
X_2(t) = \dfrac{0.32X_4^0(t-1)}{0.2X_2^0(t-1)} + 0.23|X_3|(t-2) - 0.34e^{H_1(t-3)} \\
X_3(t) = 0.1|X_1|(t-1)\cdot 0.26\sin(X_2(t))\cdot 0.4\cos(X_0(t-2)) - 0.2\cos(H_0(t-2)) \\
X_4(t) = 0.24|X_1|(t-3) - 0.43X_3^0(t) + 0.31\sin(H_0(t-3)) + 0.21H_1(t-3) \\
H_0(t) = 0.45|X_3|(t-2) \\
H_1(t) = \dfrac{0.32H_0(t-1)}{0.35e^{H_1(t-3)}\cdot 0.4X_4(t-3)}
\end{cases}
$$



RandomGraph

```
noise_gaussian = (NoiseType.Gaussian, 0, 1)
RS = RandomGraph(nvars = 5,
                 nsamples = 1500,
                 link_density = 3,
                 coeff_range = (0.1, 0.5),
                 max_exp = 2,
                 min_lag = 0,
                 max_lag = 3,
                 noise_config = noise_gaussian,
                 functions = ['','sin', 'cos', 'exp', 'abs', 'pow'],
                 operators = ['+', '-', '*', '/'],
                 n_hidden_confounders = 2)
RS.gen_equations()

d_obs_wH, d_obs = RS.gen_obs_ts()
d_obs.plot_timeseries()
```

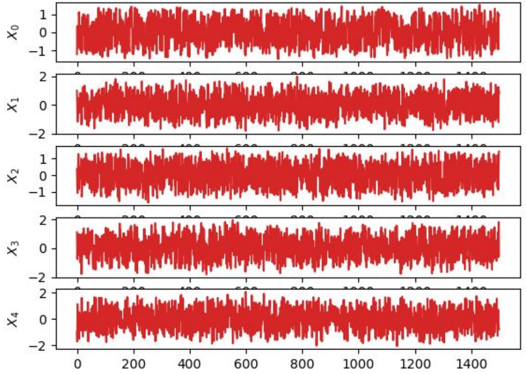
OBSERVATION



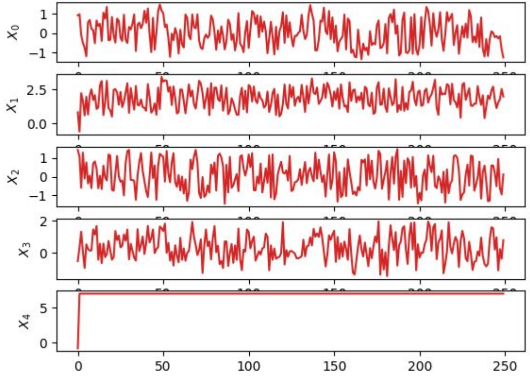
RandomGraph

```
noise_gaussian = (NoiseType.Gaussian, 0, 1)
RS = RandomGraph(nvars = 5,
                 nsamples = 1500,
                 link_density = 3,
                 coeff_range = (0.1, 0.5),
                 max_exp = 2,
                 min_lag = 0,
                 max_lag = 3,
                 noise_config = noise_gaussian,
                 functions = ['','sin', 'cos', 'exp', 'abs', 'pow'],
                 operators = ['+', '-', '*', '/'],
                 n_hidden_confounders = 2)
RS.gen_equations()

d_obs_wH, d_obs = RS.gen_obs_ts()
d_obs.plot_timeseries()

d_int = RS.intervene('X_4', 250, random.uniform(5, 10), d_obs.d)
d_int['X_4'].plot_timeseries()
```

OBSERVATION

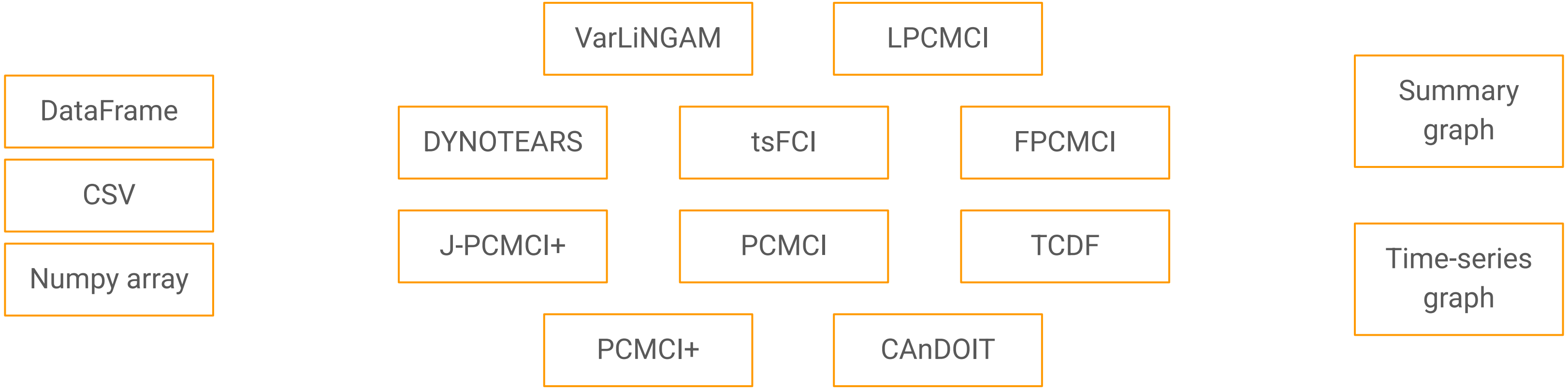


INTERVENTION



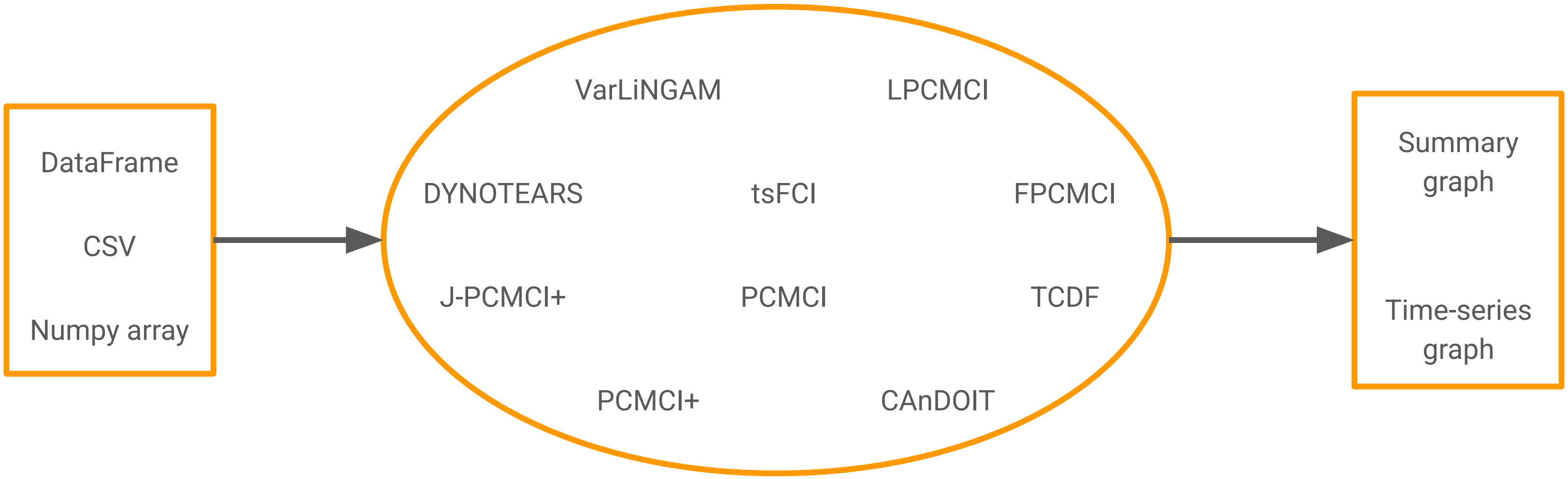
Conclusion & Future directions

- Build a unified framework for causality in time-series domain



Conclusion & Future directions

- Build a unified framework for causality in time-series domain



Future directions

- Include more causal discovery methods
- Intuitive and human-friendly interface
- Support reasoning not only discovery
  - Time-series prediction

CausalFlow
A Unified Framework for Causality in Time-Series

Personal webpage

Thank you! Questions?